Capacity-Delay-Error-Boundaries: A Composable Model of Sources and Systems

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March 09, 2015
Information vs. queueing theory

**Information theory** focuses on averages and asymptotic limits of

▶ the data rate of a source,
▶ the capacity of a channel.

It does, however, not consider delays that are due to their variability.

Unlike **queueing theory** that considers

▶ the burstiness of sources and statistical multiplexing,
▶ delays and loss that can be traded for capacity,

but assumes statistics of sources.
Outline

Basic Concepts

Additivity of the Capacity-Delay-Error Model

Composite systems

Conclusions
Network Calculus [Chang ’00]

\[ A(t) \otimes S(t) := \inf_{\tau \in [0,t]} \{ A(\tau) + S(\tau, t) \} \leq D(t) \]
Statistical network calculus [CLB ’06]
Statistical network calculus [CLB ’06]

\[\text{data} \]
Statistical network calculus [CLB ’06]
Statistical network calculus [CLB ’06]
Statistical network calculus [CLB ’06]
Statistical network calculus [LFL ’14]
Performance bounds

\[ \begin{align*}
E_A(t) & \quad B(t) \\
W(t) & \quad E_S(t)
\end{align*} \]
For a constant rate server with capacity $c$

$$E_A(t)$$

$E_A(t) := \sup_{t \geq 0} \{ E_A(t) - ct \}$

$$d_A = \frac{L_A(c)}{c}.$$
For a constant rate server with capacity $c$

$$\mathcal{L}_A(c) := \sup_{t \geq 0} \{ E_A(t) - ct \}$$
For a constant rate server with capacity $c$

\[ \mathcal{L}_A(c) := \sup_{t \geq 0} \{ E_A(t) - ct \} \]

\[ d_A = \frac{\mathcal{L}_A(c)}{c} . \]
Source

\[ d_A [s] \]

\[ c [Mb/s] \]

\[ qp = 28 \ 25 \ 22 \ 19 \]

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For a constant arrival rate $c$

$$\mathcal{L}_S(c) := \sup_{t \geq 0} \{ct - E_S(t)\}$$

$$d_S = \frac{\mathcal{L}_S(c)}{c}$$
System [ALB ’13]
Additivity of the Capacity-Delay-Error Model

\[ \mathcal{L}_A(c) := \sup_{t \geq 0} \{ E_A(t) - ct \}, \]
\[ \mathcal{L}_S(c) := \sup_{t \geq 0} \{ ct - E_S(t) \}. \]
Additivity of the Capacity-Delay-Error Model

\[ \mathcal{L}_A(c) := \sup_{t \geq 0} \{ E_A(t) - ct \}, \]
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\[ d = d_A + d_S \]
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i.e., sources and channels can be analyzed as if in isolation.
Additivitiy of the Capacity-Delay-Error Model

\[ L_A(c) := \sup_{t \geq 0} \{ E_A(t) - ct \}, \]
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i.e., sources and channels can be analyzed as if in isolation.
Composite systems

\[ c \text{ [Mb/s]} \]

\[ d_A, d_S \text{ [s]} \]

\[ qp = 28 \]

6 dB, 9, 12, 15

\[ 2, 6, 10, 14, 18 \]

\[ 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 \]
Feasible Operating Points

\begin{figure}
\centering
\includegraphics[width=\textwidth]{feasible_operating_points.png}
\caption{Feasible Operating Points}
\end{figure}
Conclusions

Legendre transforms of $E_A(t)$ and $E_S(t)$

- additive backlog and delay bounds
- permits analyzing sources and channels separately
- have the interpretation of a capacity-delay-error-tradeoff
- model includes
  - memoryless sources, Markov sources,
  - Huffman, Shannon, Lempel-Ziv coders,
  - discrete memoryless channels
  - block coding, e.g., BCH codes

- can be a step towards a unified theory ...