# Channel Estimation for OFDM Systems in case of Insufficient Guard Interval Length

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Abstract— This paper presents a novel method used to channel estimation for OFDM systems in case of insufficient guard interval length. This method suppresses additive noise and interference components by averaging the estimated channel coefficients in the time direction. The channel coefficients obtained after time averaging must be added to an adjusting coefficient to approach the true channel. This method improves the channel estimation performance significantly with the assumption that the channel is time-invariant or slowly time-varying.

Keywords—OFDM, channel estimation in presence of ISI

#### I. INTRODUCTION

In OFDM systems the channel impulse response (CIR) can be longer than the guard interval (GI). For example, in HIPERLAN/2 system [1], when the receiver moves from indoor to outdoor environment, the GI length condition is no longer met. In this case, interference distortion will appear, and the channel estimation becomes problematic.

In previous research, Yamamura and Hadara proposed in [6] the CIR estimation by using a correlator under the assumption of orthogonality between sub-carriers. However when the length of the guard interval is insufficient, the orthogonality of the sub-carriers is no longer fulfilled. Kim and Stüber [3] proposed Channel Transfer Function (CTF) estimation using a special characteristic of the pilot symbols. The inverse Fourier transformation of the pilot symbol sequence transmitted on a whole OFDM symbol gives a periodic signal sequence in time domain, where the first half of the signal sequence is considered as a part of guard interval. This can be interpreted as a lengthening of the guard interval. Thus the ISI distortion might not occur on the received pilot symbol. This method, however, is only applicable, when the two following conditions are met:

The OFDM symbols, in which the pilot symbols are located, must be reserved completely for pilot transmission. The CIR length must be shorter than the guard length

plus half of the length of one OFDM duration. The basis for the method developed in this paper can be

found in [2], in which the OFDM system is performed under the condition of sufficient guard interval length. However, in case of insufficient guard interval length, the Intersymbol Interference (ISI) and the Inter-Carrier Interference Caused by the Insufficient Guard interval length (ICI-CIG) [5], and together with the additive noise are present in the estimated CTF. These distortions can be suppressed by averaging the estimated CTF. But the result obtained by averaging the estimated CTF is not the final result. The reason is that the part of the CIR outside the GI is attenuated by the averaging process. This attenuation must be compensated by adding an appropriate adjusting coefficient to the corresponding part of the CTF. By this method, the MSE of the estimated CTF is significantly reduced, approximately 20 dB with the same Signal-to-Noise Ratio (SNR).

The organization of this paper is as follows: The influences of interference distortions on the received pilot symbols are studied in section II. Section III presents the channel estimator by averaging the consecutive CTFs in time direction. Section IV introduces the performance of channel estimation in terms of MSE for different characteristics of the pilot symbol. Finally, section V concludes the paper.

# II. RECEIVED PILOT SYMBOLS WITH INSUFFICIENT GUARD LENGTH

As introduced in [5], if the guard interval condition is too short, then ISI and ICI-CIG distortions will occur. Observing the received pilot symbol on *p*-th sub-carrier and on *i*-th OFDM symbol, similar to the demodulated data symbol according to Eq. (8) in [5], the received pilot symbol  $\hat{R}_{p,i}$  can also be decomposed as

$$\hat{R}_{p,i} = \hat{R}_{p,i}^{U} + \hat{R}_{p,i}^{\text{ICI-CIG}} + \hat{R}_{p,i}^{\text{ICI-CTC}} + \hat{R}_{p,i}^{\text{ISI}}, \qquad (1)$$

where  $\hat{R}_{p,i}^{\text{U}}, \hat{R}_{p,i}^{\text{ICI-CIG}}, \hat{R}_{p,i}^{\text{ICI-CTC}}$  and  $\hat{R}_{p,i}^{\text{ISI}}$  are the useful term, the ICI caused by insufficient guard length, the ICI caused by the time variation of the channel and the ISI term introduced on the received pilot symbol, respectively. According to Eq. (18) in [5], the useful term  $\hat{R}_{p,i}^{\text{U}}$  can be written:

$$\hat{R}_{p,i}^{\rm U} = S_{p,i} \bigg\{ H_1(p) + \alpha H_2(p) + \eta_p \bigg\}$$
(2)

where  $H_1(p)$  and  $H_2(p)$  are the CTF of the first truncated channel  $h_1(k)$ , k = 0, ..., G - 1, and the second truncated channel  $h_2(k)$ ,  $k = G, ..., N_{\text{FFT}} - 1$ , which are already defined in [5]. G and  $N_{\text{FFT}}$  are the guard interval length and the FFT length;  $h_1(k)$  is the first part of h(k) inside the guard interval, and  $h_2(k)$  belongs to the second part of h(k) outside the guard interval. The multiplicative distortion  $\alpha$  and the additive distortion  $\eta_p$  are given in Eqs. (15) and (17) [5], respectively <sup>1</sup>.  $S_{p,i}$  is the transmitted pilot symbol on the *p*-th sub-carrier and the *i*-th OFDM symbol. To obtain the expression of the last terms in Eq. (1), we define the transmitted data vector  $\vec{d_i}$  and the transmitted pilot symbol vector  $\vec{S_i}$  as follows:

$$d_{i} = [0, d_{1,i}, ..., d_{D_{f}-1,i}, 0, d_{D_{f}+1,i}, ..., , ..., d_{N_{C}-2,i}, 0]$$
  
$$\vec{S}_{i} = [S_{0,i}, 0, ..., 0, S_{D_{f},i}, 0, ..., 0, S_{N_{C}-1,i}], \quad (3)$$

where  $d_{n,i}$  is the data symbol on the *n*-th sub-carrier and the *i*-th OFDM symbol. The pilot symbols are periodically assigned on some sub-carriers with the pilot distance  $D_f$ . For convenience, the expression of  $\hat{R}_{p,i}^{\text{ICI-CIG}}$  in [5] is given as follows:

$$\hat{R}_{p,i}^{\text{ICI-CIG}} = \sum_{n=0,n\neq p}^{N_{\text{C}}-1} (d_{n,i} + S_{n,i}) \Biggl\{ \frac{1}{N_{\text{FFT}}} \Biggl[ \\ \sum_{t_{d}=0}^{N_{\text{P}}-G-1} \sum_{k=G}^{t_{d}+G} h_{2}(k) e^{\frac{-j2\pi nk}{N_{\text{FFT}}}} e^{\frac{j2\pi (n-p)t_{d}}{N_{\text{FFT}}}} \\ + \sum_{t_{d}=N_{\text{P}}-G}^{N_{\text{C}}-1} H_{2}(n) e^{\frac{j2\pi (n-p)t_{d}}{N_{\text{FFT}}}} \Biggr] \Biggr\}, \quad (4)$$

where the denotations  $n, p, N_{\rm T} = G + N_{\rm FFT}, t_d, k, N_{\rm P}$ are the sub-carrier index, the observed sub-carrier index, the total OFDM block length including the guard interval, the time index, the tap index and the tap number of the CIR, respectively. The term in the parentheses of Eq. (4) is denoted  $\mathcal{A}(n, p)$ . Then, the following term:

$$\hat{H}_{n,p}^{\text{ICI-CIG}} = \begin{cases} \frac{1}{N_{\text{FFT}}} \mathcal{A}(n,p), & \text{if } n \neq p\\ 0, & \text{otherwise} \end{cases}$$
(5)

is indicated as the ICI-CIG coefficient. By denoting ICI-CIG vector as follows:

$$\vec{H}_{p}^{\text{ICI-CIG}} = [H_{0,p}^{\text{ICI-CIG}}, ..., H_{n,p}^{\text{ICI-CIG}}, ..., \\ , ..., H_{N_{\text{C}}-1,p}^{\text{ICI-CIG}}],$$
(6)

 $\hat{R}_{p,i}^{\text{ICI-CIG}}$  can be expressed as

$$\hat{R}_{p,i}^{\text{ICI-CIG}} = (\vec{d}_i + \vec{S}_i) \vec{H}_p^{\text{ICI-CIG}}$$
(7)

The contribution of  $\hat{R}_{p,i}^{\text{ICI-CTC}}$  in Eq. (1) can be ignored, because we consider only the case of a time-invariant or a slowly time-varying channel. According to Eq. (27) in [5], the contribution of  $\hat{R}_{p,i}^{\text{ISI}}$  in Eq. (1) is rewritten as follows:

$$\hat{R}_{p,i}^{\text{ISI}} = \sum_{n=0}^{N_{\text{C}}-1} (d_{n,i-1} + S_{n,i-1}) \bigg\{ \frac{1}{N_{\text{FFT}}} \sum_{t_d=0}^{N_{\text{P}}-1} \sum_{k=G+t_d}^{N_{\text{P}}-1} h_2(k) e^{\frac{-j2\pi nk}{N_{\text{FFT}}}} e^{\frac{j2\pi [(n-p)t_d+nN_{\text{T}}]}{N_{\text{FFT}}}} \bigg\}.$$
 (8)

 ${}^{1}\alpha = 1, \eta_p = 0$  for the case of sufficient guard interval length.

The term in brackets in Eq. (8) is the ISI coefficient and is denoted  $H_{n,p}^{\text{ISI}}$ . We can represent the ISI coefficients as an ISI vector

$$\vec{H}_{p}^{\text{ISI}} = [H_{0,p}^{\text{ISI}}, ..., H_{n,p}^{\text{ISI}}, ..., H_{N_{\text{C}}-1,p}^{\text{ISI}}], \qquad (9)$$

then  $\hat{R}_{p,i}^{\text{ISI}}$  is given as follows:

$$\hat{R}_{p,i}^{\text{ISI}} = (\vec{d}_{i-1} + \vec{S}_{i-1})\vec{H}_p^{\text{ISI}}.$$
(10)

Summation of  $\hat{R}_{p,i}^{\text{U}}$ ,  $\hat{R}_{p,i}^{\text{ICI-CIG}}$  and  $\hat{R}_{p,i}^{\text{ISI}}$  in Eqs (2), (7) and (10) gives the expression of the received pilot symbols as follows:

$$\hat{R}_{p,i} = S_{p,i}[H_1(p) + \alpha H_2(p) + \eta_p] + (\vec{d}_i + \vec{S}_i)\vec{H}_p^{\text{ICI-CIG}} + (\vec{d}_{i-1} + \vec{S}_{i-1})\vec{H}_p^{\text{ISI}}.$$
(11)

The contributions of  $\hat{R}_{p,i}^{\text{ISI}}$  and  $\hat{R}_{p,i}^{\text{ICI-CIG}}$  are highly dependent on the characteristics of the pilot symbols. In the following, the influence of the characteristics of pilot symbols on its interference contributions is studied in detail.

## A. Constant pilot symbols

In this case, the pilot symbol is simply a constant factor,  $S_{p,i} = S$ ,  $\forall p$ . With the definition of  $\vec{H}_p^{\text{ISI}}$  in Eq. (9), we can write the ISI contribution derived from the transmitted pilot symbols in the received pilot symbol as follows:

$$\vec{S}_{i-1}\vec{H}_{p}^{\text{ISI}} = \frac{S}{N_{\text{FFT}}} \sum_{m=0}^{L_{f}-1} \sum_{t_{d}=0}^{N_{\text{P}}-G-1} \sum_{k=G+t_{d}}^{N_{\text{P}}-1} h_{2}(k) \\ e^{\frac{-j2\pi m D_{f}k}{N_{\text{FFT}}}} e^{\frac{j2\pi [(mD_{f}-p)t_{d}+mD_{f}N_{\text{T}}]}{N_{\text{FFT}}}}$$
(12)

where  $L_f$  is the number of pilot symbols per OFDM symbol and defined by  $L_f = \lceil N_{\rm C}/D_f \rceil$ . The operation  $\lceil x \rceil$  denotes the smallest integer larger or equal to x. Equation (12) is deduced from Eq. (8), in which the sub-carrier index p is substituted by the sub-carrier index  $mD_f$  where the pilot symbols are situated. We suppose that the number of sub-carriers  $N_{\rm C} = N_{\rm FFT}$  and  $N_{\rm C}$  is divisible by the pilot distance  $D_f$ . Then the right-hand side of Eq. (12) can be rearranged as

$$\vec{S}_{i-1}\vec{H}_{p}^{\text{ISI}} = \frac{S}{N_{\text{FFT}}} \sum_{t_{d}=0}^{N_{\text{P}}-G-1} \sum_{k=G+t_{d}}^{N_{\text{P}}-1} h_{2}(k) \\ \times \left\{ \sum_{m=0}^{L_{f}-1} e^{\frac{-j2\pi m(k-t_{d}+N_{\text{T}})}{L_{f}}} \right\} e^{\frac{-j2\pi pt_{d}}{N_{\text{FFT}}}} (13)$$

It can be easily seen that the term in brackets of Eq. (13) is zero, thus Equation (10) becomes

$$\hat{R}_{p,i}^{\mathrm{ISI}} = \vec{d}_{i-1} \vec{H}_p^{\mathrm{ISI}} \tag{14}$$

Equation (14) reveals that the ISI contribution introduced by the constant pilot symbols completely vanishes. The impact of transmitted pilot symbols on ICI-CIG contribution is demonstrated in Eq. (11), where the ICI-CIG contribution caused by the transmitted pilot symbols is  $\vec{S}_i \vec{H}_p^{\text{ICI-CIG}}$  which can be expanded for constant pilot symbol as follows:

$$\vec{S}_{i}\vec{H}_{p}^{\text{ICI-CIG}} = \sum_{n=0}^{N_{C}-1} SH_{n,p}^{\text{ICI-CIG}}$$

$$= \frac{1}{N_{\text{FFT}}} \sum_{n=0}^{N_{C}-1} S \left\{ \sum_{t_{d}=0}^{N_{P}-G-1} \sum_{k=G}^{t_{d}+G} h_{2}(k) \right\}$$

$$\times e^{-j2\pi n(k-t_{d})/N_{\text{FFT}}} e^{-j2\pi p t_{d}/N_{\text{FFT}}}$$

$$+ \sum_{t_{d}=N_{P}-G}^{N_{C}-1} \left[ \sum_{n=0}^{N_{C}-1} H_{2}(n) e^{j2\pi n t_{d}/N_{\text{FFT}}} \right]$$

$$\times e^{-j2\pi p t_{d}/N_{\text{FFT}}} \left\} - S[\alpha H_{2}(p) + \eta_{p}]. \quad (15)$$

After some manipulations, equation (15) can be simplified as follows:

$$\vec{S}_i \vec{H}_p^{\text{ICI-CIG}} = SH_2(p) - S[\alpha H_2(p) + \eta_p].$$
 (16)

The expression of the received pilot symbol in Eq. (11) in connection with results obtained in Eq. (14) and Eq. (16) can be simplified with  $H(p) = H_1(p) + H_2(p)$  as follows:

$$\hat{R}_{p,i} = SH(p) + \vec{d}_i \vec{H}_p^{\text{ICI-CIG}} + \vec{d}_{i-1} \vec{H}_p^{\text{ISI}}.$$
 (17)

From result of Eq. (17), it is to conclude that, for the case of constant pilot symbols, the received pilot symbol is impaired only by transmitted data symbols. The first term of Eq. (17) describes the product of the pilot symbol with the associated channel coefficient. The last two terms can be considered as additive noise and denoted as  $\hat{R}_{p,i}^{C}$ . Finally, Equation (17) is rewritten as:

$$\hat{R}_{p,i} = SH(p) + \hat{R}_{p,i}^{C}.$$
 (18)

# B. Pilot symbols with pseudo-random phase

Pilot symbols with pseudo-random phase can be given as

$$S_{p,i} = S e^{j\varphi_{p,i}}, \tag{19}$$

where  $\varphi_{p,i}$  is a pseudo-random process which is evenly distributed in the range of  $[-\pi, \pi]$ . In this case, the data sequence is a random process, and the pilot sequence is a pseudo-random process, as well. Both introduce its interference distortions in the received pilot symbol. Hence, Equation (11) is rewritten as

$$\hat{R}_{p,i} = S_{p,i} \left\{ H_1(p) + \alpha H_2(p) + \eta_p \right\} + \hat{R}_{p,i}^{\text{ICI-CIG}} \\
+ \hat{R}_{p,i}^{\text{ISI}}.$$
(20)

The last two terms in Eq. (20) are considered as distortion and are denoted as  $\hat{R}_{p,i}^{\text{R}}$ .

# III. PROPOSED CHANNEL ESTIMATION METHOD

For a large number of sub-carriers, the central limit theorem can be invoked and the ICI-CIG and ISI contributions caused by the transmitted data symbols can be treated like additive noise. For constant pilot symbols,  $R_{p,i}^{\rm C}$  can be considered as additive noise with zero-mean which is also the case for  $\hat{R}_{p,i}^{\mathrm{R}}$  with the pseudo-random pilot symbols. This is the key point to start with a new channel estimation method. In literature, Kang and Song [2] have considered an OFDM system as a set of parallel Gaussian channels with different attenuation factors for each sub-carrier. This is true for a time-invariant channel. The frequency response of the multi-path channel is estimated by time averaging the consecutive estimated channel coefficients, which are obtained by dividing the received pilot symbols by the known symbols. This method is considered in this paper in order to suppress the ISI and ICI-CIG distortions. Unlike additive noise, the ISI and ICI-CIG can be derived from the transmitted data. the pilot symbol and also the second truncated channel. Thus, the transmitted data and also the pilot symbol affect mutually its contributions in the ISI and ICI-CIG distortions. Since the transmitted data is random and unknown, their contributions in the ISI and ICI-CIG distortions are not avoidable. However, because the transmitted pilot symbols are known symbols, they affect ISI and ICI-CIG distortions differently depending on their characteristics. A new channel estimation algorithm to suppress ISI and ICI-CIG distortions proceeds in three steps as follows: 1. In the first step, an initial estimated CTF is obtained by dividing the received pilot symbol  $\hat{R}_{p,i}$  by the known symbol  $S_{p,i}$ :

$$\hat{H}_{p,i} = \frac{\hat{R}_{p,i}}{S_{p,i}}.$$
(21)

2. In the second step, an averaged estimated CTF is obtained by averaging the first estimated CTF over  $L_a$ OFDM symbols in the time direction. This can be done under the assumption that the channel coefficients are constant over the averaging range:

$$\bar{H}(p) = \frac{\sum_{i=0}^{L_a - 1} \hat{H}_{p,i}}{L_a}.$$
(22)

The averaged estimated CTF  $\overline{H}(p)$  will be adjusted according to which characteristic of the transmitted pilot symbols are used. The adjusted factor will be discussed in the following subsection.

3. After averaging the CTFs at the positions of the pilot symbols, the CTF at the positions of the data symbols are obtained by interpolation.

The ISI and ICI-CIG distortions highly depend on the characteristics of the pilot symbols, and so does the performance of this algorithm.

# A. Applied for constant pilot symbols

From Eqs. (18) and (21), the first estimated CTF for the case of constant pilot symbols is:

$$\hat{H}_{p,i} = H(p) + \frac{\hat{R}_{p,i}^{C}}{S}.$$
(23)

According to Eq. (22), the second estimated CTF is

$$\bar{H}(p) = \frac{\sum_{i=0}^{L_a - 1} \left( H(p) + \hat{R}_{p,i}^{\rm C} / S \right)}{L_a} = H(p) + \frac{\sum_{i=0}^{L_a - 1} \hat{R}_{p,i}^{\rm C}}{L_a \cdot S}.$$
(24)

Since  $\hat{R}_{p,i}^{C}$  can be treated as a Gaussian process with zeromean, the averaged value of  $\hat{R}_{p,i}^{C}$  over a sufficient length of  $L_a$  OFDM symbols will approach zero. What is obtained after averaging is close to the true channel transfer function. It can be found that the stronger the pilot symbols are boosted, the more the interference distortion will be reduced. On the one hand, the constant pilot symbols turn out to be the optimal characteristic on behalf of interference distortion suppression. On the other hand, as stated in [4], the constant pilot symbols lead to an extremely high crest factor. In order to reduce the influence of pilot symbols on the crest factor of an OFDM signal, generally pilot symbols with pseudo-random phase are favoured.

#### B. Applied for pseudo-random pilot symbols

With  $\hat{R}_{p,i}$  being last two terms in (20), equation (21) yields

$$\hat{H}_{p,i} = H_1(p) + \alpha H_2(p) + \eta_p + \frac{\hat{R}_{p,i}^{\mathrm{R}}}{S_{p,i}}.$$
 (25)

Comparing (25) with (23), it is to find that the first three terms of (25) are not the true channel coefficient. Furthermore, the distortion component  $\hat{R}_{p,i}^{\rm C}$  in (23) stems merely from the transmitted data symbols, whereas the distortion component  $\hat{R}_{p,i}^{\rm R}$  in (25) results not only from the transmitted pilot symbols. After performing the second step of channel estimation, the averaged channel coefficients become

$$\bar{H}(p) = H_1(p) + \alpha H_2(p) + \eta_p + \frac{\sum_{i=0}^{L_a - 1} \hat{R}_{p,i}^{\mathrm{R}} / S_{p,i}}{L_a}.$$
 (26)

The last term of (26) is distortion appearing in the estimated channel. This term vanishes when the averaging range is long enough as proved in the following:

$$\lim_{L_a \to \infty} \frac{\sum_{i=0}^{L_a} \hat{R}_{p,i}^{\mathrm{R}} / S_{p,i}}{L_a} = E\{\frac{\hat{R}_{p,i}^{\mathrm{R}}}{S_{p,i}}\} = S^{-1}E\{\hat{R}_{p,i}^{\mathrm{R}}e^{-j\varphi_{p,i}},\}$$
(27)

where  $S_{p,i}$  has pseudo-random phase as given in (19), in which  $\varphi_{p,i}$  is a pseudo-random process. As the pilot symbols and the data symbols are statistically independent, the pilot symbols and the interference distortion  $\hat{R}_{p,i}^{\text{R}}$  are statistically independent, too. Equation (27) can be solved as follows:

$$\lim_{L_a \to \infty} \frac{\sum_{i=0}^{L_a - 1} \hat{R}_{p,i}^{\mathrm{R}} / S_{p,i}}{L_a} = S^{-1} E\{\hat{R}_{p,i}^{\mathrm{R}}\} E\{e^{-j\varphi_{p,i}}\}.$$
 (28)

Since  $\hat{R}_{p,i}^{\text{R}}$  is a random variable having zero-mean,  $E\{\hat{R}_{p,i}^{\text{R}}\} = 0$  is valid. The expectation value  $E[e^{-j\varphi_{p,i}}]$  must be limited, because  $e^{-j\varphi_{p,i}}$  has amplitude of one. Therefore, equation (28) can be rewritten as follows:

$$\lim_{L_a \to \infty} \frac{\sum_{i=0}^{L_a - 1} \hat{R}_{p,i}^{\mathrm{R}} / S_{p,i}}{L_a} = 0.$$
 (29)

In the case of a very long averaging range  $L_a$ , the last term of (27) vanishes. In order to have the final result close to the true channel, an adjusting coefficient  $\gamma_p$  defined as follows:

$$\gamma_p = \frac{1}{N_{\rm FFT}} \sum_{t_d=0}^{N_{\rm P}-G-1} \sum_{k=G+t_d+1}^{N_{\rm P}-1} h_2(k) e^{-j2\pi pk/N_{\rm FFT}}, \quad (30)$$

must be added to the first three terms of (27). It is easy to prove that

$$H_1(p) + \alpha H_2(p) + \eta_p + \gamma_p = H(p).$$
 (31)

However,  $h_2(k)$  in (30) is unkown. Hence,  $h_2(k)$  is replaced by  $\bar{h}_2(k)$ , where  $\bar{h}_2(k)$  is the second truncated channel, which is obtained from the averaged channel coefficient  $\bar{H}(p)$  as follows: First, take the IFFT of  $\bar{H}(p)$ . That is  $\bar{h}(q) = \text{IFFT}[\bar{H}(p)]$ . Second, set the first term within the guard interval of  $\bar{h}(q)$  to zero.

#### IV. SIMULATION RESULTS

The OFDM parameters used for simulation are adopted from HIPELAN/2 specified in [1]. The indoor channel model used for simulations is described in [5]. Using these parameters, we evaluate the performance of the proposed channel estimator for the system in the presence of ISI and ICI-CIG in terms of Mean Square Error (MSE). Figure (1) demonstrates the comparison results obtained by the proposed channel estimator using constant pilot symbols and pilot symbols with pseudo-random phase for the case of a time-invariant channel without guard interval. It is impressive to see that, increasing the length of time averaging  $L_a$ , the MSE obtained by the proposed channel estimator is dramatically reduced. The longer the length of time averaging  $L_a$ , the better the result that can be achieved. It is important to note that the proposed channel estimator using constant pilot symbols shows better results than using pilot symbols with pseudo-random phase. This can be explained by comparing Eqs. (24) and (26).



Fig. 1. MSE obtained for a time-invariant channel without a guard interval.



Fig. 2. Comparison of MSE obtained by the proposed method with conventional method for an OFDM system without GI and on a time-invariant channel.

The first term of Eq. (24) is the true channel, whereas the first three terms of Eq. (26) do not represent the true channel, because the adjusting coefficient as described in Eq. (31) must be added to get the true CTF. Moreover, the second term of Eq.(24) regarded as noise stems only from the data sequence, whereas the last term of Eq. (26)regarded as noise is conducted not only from the data sequence but also the training sequence. The results in Fig. 2 confirms the gain of the proposed method, where the pilot symbols with pseudo-random phase are used for simulations. Comparing the proposed method with the conventional method (without averaging and adjusting), it is to see that 20 dB of MSE is improved in the same SNR. The results for the case of a time-variant channel are shown in Fig. 3. The MSE can only be reduced if the averaging range is suitable for the given Doppler frequency. Varying the length of averaging range gives the



Fig. 3. MSE obtained for slowly time-varying channel.

following results: The MSE is reduced when the channel can be assumed to be constant in the time averaging interval. The MSE is increased on the other hand when the averaging range is very long so that the channel is no longer constant in this interval.

#### V. CONCLUSION

Even in OFDM systems suffering from ISI, the CTF can be accurately obtained by averaging over a number of samples under the assumption that the channel is timeinvariant or slowly varying over the time averaging interval. The advantage of this method is that no prior information of the channel and no significant computation are required.

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