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***Abstract*—Non-line-of-sight propagation in wireless localization systems leads to a challenging problem for the estimation of a mobile position. A hybrid idea has been considered as a promising approach to increase estimation accuracy. Existing methods, however, are formulated using separate measurements, necessitating different observation quantities, e.g. received signal and received signal strength, thus making the parameter estimation cumbersome. In this paper, we propose a new hybrid wireless geolocation model requiring only one observation quantity, namely the received signal. The attenuation model for transmit power from base stations is explored herein to capture propagation features in the received signal model. Fortunately, it also provides a more realistic approach to wireless geolocation. To investigate geolocation accuracy, the Cramér-Rao bound (CRB) is derived for the estimation error of the mobile position. For any value of path loss exponent, the obtained result provides a generalized form of the CRB for the usual time delay case. In small cells, e.g. office building picocells, numerical examples illustrate that the accuracy of mobile position estimation exploring path loss is improved comparing with that provided by the usual time delay method.**

***Index Terms*—Parameter estimation, non-line-of-sight propagation, path loss.**

I. Introduction

Recently, the CRB has been analyzed in [1] for several geolocation schemes in the presence of non-line-of-sight (NLoS). A common result is revealed that the Fisher information matrix (FIM) of the hybrid scheme using signal strength (SS) and time-difference of arrival (TDoA) can be acquired by the superposition of the FIMs from both schemes. Hybrid schemes outperform those using only one feature in the aspects of estimation accuracy [2] and reliability [3]. It is fruitful to note that the frameworks in [1] and [2] are composed of two separate techniques, which require two different measurements, e.g. baseband received signal and mean signal strength, respectively. Even though both measures are jointly formulated via the positioning distance, the parameter estimation needs to collect two kinds of observation data and then be performed separately. This kind of combination thus makes the parameter estimation cumbersome.

In this paper, we consider the inherent accuracy of a mobile station (MS) position estimation in a handset-based mutilateral geolocation system using time-of-arrival (ToA). A path attenuation model is incorporated into the time delay model. This leads to a more realistic investigation of wireless geolocation. Since the path loss is an attenuation feature of the SS, the presented framework can be referred to as a hybrid SS/ToA method. The channel amplitude is shown to be explicitly related to path attenuation. Unlike [1] and [2], the signal strength model and the time delay model are herein combined together to form a composite received signal model. As a consequence, it requires only the received signal for performing the wireless NLoS geolocation. To investigate the accuracy of position estimation, the corresponding CRB is derived for the estimation error of the mobile position. This result gives a generalized form of the CRB with respect to the usual time delay case. Numerical examples in small cells, e.g. in office building picocells in the same floor, illustrate that when the path attenuation for any value of path loss exponent is taken into account, the accuracy of the mobile position estimation is improved comparing with the usual time delay method. The performance improvement is significant, particularly in the system that 1) invokes the signal with a small effective bandwidth, 2) has a small distance between the mobile and base station, and 3) has a high value of the path loss exponent, i.e. high path attenuation.

II. System Model

Let us consider a MS transmitting a radio signal through a wireless channel to a number of base stations (BSs). Let B be the number of all BSs, whose locations, $\mathbf{p}_b = \begin{bmatrix} x_b & y_b \end{bmatrix}^{\mathrm{T}}$; $b \in \{1, 2, \ldots, B\}$, are known. We assume that there is no loss of energy for the transmitted signal when radio waves propagate in a media. There is, however, attenuation by the channel. At each base station, the received energy at the b-th BS can be expressed by (see e.g. [4, p. 46] and [5, p. 38])

$$E_b = \kappa \frac{d_0^{\gamma_b}}{d_b^{\gamma_b}} E_{\mathrm{s}}, \tag{1}$$

where d_0 is the close-in reference in the far field region, d_b is the distance between the MS and the b-th BS, γ_b is the path loss exponent at the b-th BS, $E_{\mathrm{s}} = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ is the energy of a transmitted signal $s(t)$, and κ is the unitless constant depending on antenna characteristics and average channel attenuation given by

$$\kappa = \frac{c^2}{16\pi^2 f_0^2 d_0^2}, \tag{2}$$

with the center frequency f_0 and the speed of light c. Assume that the discrimination between LoS and NLoS has been conducted (see e.g. [6]–[9] and references therein). Let $M < B$

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be the number of BSs that receive a set $\{1,2,\ldots,M\}$ of NLoS signals. The received signal amplitudes $\{a_b\}_{b=1}^{B}$ and the positive delay distances $\{l_m\}_{m=1}^{M}$ are assumed to be unknown, whereas the position of mobile station, $\mathbf{p}=\begin{bmatrix}x & y\end{bmatrix}^{\mathrm{T}}$, is the parameter of interest. Let τ_b be the time delay of received signal at the b-th BS:

$$\tau_b(x,y,l_b)=\frac{1}{c}\left(\sqrt{\tilde{x}_b^2+\tilde{y}_b^2}+l_b\right), \tag{3}$$

where $\tilde{x}_b=x-x_b$, and $\tilde{y}_b=y-y_b$, $l_b=0$; for $b\in\{M+1,M+2,\ldots,B\}$. As $d_b=c\tau_b$, the noiseless energy based on (1) can be rewritten as

$$E_b=\kappa\frac{1}{\left(\sqrt{\left(\frac{\tilde{x}_b}{d_0}\right)^2+\left(\frac{\tilde{y}_b}{d_0}\right)^2}+\frac{l_b}{d_0}\right)^{\gamma_b}}E_{\mathrm{s}}. \tag{4}$$

Since (1) and (4) are valid only in the far field, it is assumed that d_0 is less than $\sqrt{\tilde{x}_b^2+\tilde{y}_b^2}$. This means that within a circle of radius d_0 there are no BSs. The received baseband signal can be written as [1]

$$r_b(t)=a_b s(t-\tau_b)+n_b(t), \tag{5}$$

where $s(t)$ is the known waveform, a_b and τ_b are the amplitude and time delay of the propagation to the b-th BS, and $n_b(t)$ is an additive noise at the b-th BS and assumed to be a complex-valued white Gaussian process with zero mean and variance σ_{n}^2.

Since $E_b=a_b^2E_{\mathrm{s}}$, the unitless amplitude is given by

$$a_b=\sqrt{\kappa}\frac{1}{\left(\sqrt{\left(\frac{\tilde{x}_b}{d_0}\right)^2+\left(\frac{\tilde{y}_b}{d_0}\right)^2}+\frac{l_b}{d_0}\right)^{\frac{1}{2}\gamma_b}}. \tag{6}$$

Note that when there is no path attenuation ($\gamma_b=0$), the amplitude becomes $a_b=\sqrt{\kappa}=\frac{c}{4\pi f_0 d_0}$. Assume that the transmitted signal is nonzero over the interval $[0,T_{\mathrm{s}}]$, where T_{s} is the signal period.

In this model, we can see that $r_b(t)$ is a random signal due to the randomness of $n_b(t)$. Since the position $\mathbf{p}$ and the nuisance parameter l_b are unknown and deterministic, their reparameterizations a_b and τ_b are as well. All the unknown parameters can be aggregated into the vector $\boldsymbol{\theta}\in\mathbb{R}^{(M+2)\times 1}$ as

$$\boldsymbol{\theta}=\begin{bmatrix}\mathbf{p}^{\mathrm{T}} & \mathbf{l}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}, \tag{7}$$

where $(\cdot)^{\mathrm{T}}$ is the transpose and $\mathbf{l}\in\mathbb{R}^{M\times 1}$ is given by

$$\mathbf{l}=\begin{bmatrix}l_1 & l_2 & \cdots & l_M\end{bmatrix}^{\mathrm{T}}. \tag{8}$$

Let the solution of the homogeneous Fredholm integral equation

$$\int_0^T \varphi_b(t,\acute{t})f_{b,k}(\acute{t})\mathrm{d}\acute{t}=\lambda_{b,k}f_{b,k}(t)\quad ;k\in\{1,2\ldots,K\}, \tag{9}$$

be the eigenvalue $\lambda_{b,k}$ and the orthonormal function $f_{b,k}(t)$, where T is the observation period, K is the number of basis functions, the kernel $\varphi_b(t,\acute{t})$ is the eigenfunction, which is the noise auto covariance function. According to the Karhunen-Loève (KL) expansion (see e.g. [10, p. 37], [11, p. 279], and [12, p. 298]), the signal can be sampled from $f_{b,k}(t)$ as

$$r_b(t)=\lim_{K\to\infty}\sum_{k=1}^{K}r_{b,k}f_{b,k}(t), \tag{10}$$

where the received signal sample is given by $r_{b,k}=\int_0^T f_{b,k}(t)r_b(t)\mathrm{d}t$. From (5), the received signal sample can be expressed by

$$r_{b,k}=a_b s_{b,k}+n_{b,k}, \tag{11}$$

where the signal and noise samples are given by $s_{b,k}=\int_0^T f_{b,k}(t)s(t-\tau_b)\mathrm{d}t$ and $n_{b,k}=\int_0^T f_{b,k}(t)n_b(t)\mathrm{d}t$. Assume that the basis function $f_{b,k}(t)$ is chosen such that the noise samples $\{n_{b,k}\}_{k=1}^{K}$ are identically and independently distributed. The probability density function (PDF) of the complex Gaussian multivariate $\{r_{b,k}\}_{k=1}^{K}$ can be written as

$$p(r_{b,1},\ldots,r_{b,K}|\tau_b)=\frac{1}{(\pi\sigma_{\mathrm{n}}^2)^K}\mathrm{e}^{-\frac{1}{\sigma_{\mathrm{n}}^2}\sum_{k=1}^{K}|r_{b,k}-a_b s_{b,k}|^2}. \tag{12}$$

Given the continuous signal $r_b(t);\,t\in(0,T]$, the likelihood of τ_b can be written in logarithm scale as

$$\begin{aligned}\ell(\tau_b|r_b(t);t\in(0,T])&=\lim_{K\to\infty}\ln\left(p(r_{b,1},\ldots,r_{b,K}|\tau_b)\right)\\&\doteq-\frac{1}{\sigma_{\mathrm{n}}^2}\int_0^T|r_b(t)-a_b s(t-\tau_b)|^2\mathrm{d}t,\end{aligned} \tag{13}$$

where $\ln(\cdot)$ is the natural logarithm function and $\doteq$ is the equivalence due to neglecting an irrelevant term. Assume that the noise is independent of each other. Given the received signal of all base stations $\mathbf{r}(t)\in\mathbb{C}^{B\times 1}$

$$\mathbf{r}(t)=\begin{bmatrix}r_1(t) & r_2(t) & \cdots & r_B(t)\end{bmatrix}^{\mathrm{T}}, \tag{14}$$

the log-likelihood function can be derived from

$$\begin{aligned}\ell(\boldsymbol{\tau}|\mathbf{r}(t);\,t\in(0,T])&=\lim_{K\to\infty}\ln\left(p(\mathbf{r}[1],\ldots,\mathbf{r}[K]|\boldsymbol{\tau})\right)\\&\doteq-\frac{1}{\sigma_{\mathrm{n}}^2}\sum_{b=1}^{B}\int_0^T|r_b(t)-a_b s(t-\tau_b)|^2\mathrm{d}t,\end{aligned} \tag{15}$$

where $\boldsymbol{\tau}\in\mathbb{R}^{B\times 1}$ is defined by

$$\boldsymbol{\tau}=\begin{bmatrix}\tau_1 & \tau_2 & \cdots & \tau_B\end{bmatrix}^{\mathrm{T}}, \tag{16}$$

and $\mathbf{r}[k]\in\mathbb{C}^{B\times 1}$ is given by

$$\mathbf{r}[k]=\begin{bmatrix}r_{1,k} & r_{2,k} & \cdots & r_{B,k}\end{bmatrix}^{\mathrm{T}}. \tag{17}$$

III. Cramér-Rao Lower Bound

Let $\hat{\boldsymbol{\theta}}$ be any unbiased estimate of $\boldsymbol{\theta}$. Then, the accuracy of $\hat{\boldsymbol{\theta}}$ is bounded by the Cramér-Rao inequality:

$$\mathrm{E}\{(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})^{\mathrm{T}}\}\succeq\mathbf{J}_{\boldsymbol{\theta}\boldsymbol{\theta}}^{-1}, \tag{18}$$

where $\mathrm{E}\{\cdot\}$ is the expectation with respect to $p(\mathbf{r}(t);\,t\in(0,T]|\boldsymbol{\theta})$, $\mathbf{J}_{\boldsymbol{\theta}\boldsymbol{\theta}}=-\mathrm{E}\left\{\frac{\partial^2}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^{\mathrm{T}}}\ln(p(\mathbf{r}(t);\,t\in(0,T]|\boldsymbol{\theta}))\right\}$ is the FIM, $(\cdot)^{-1}$ is the inverse operator, and $\mathbf{B}\succeq\mathbf{C}$ means that the matrix $\mathbf{D} = \mathbf{B} - \mathbf{C}$ is positive semidefinite. Let us define the total positioning accuracy of both axes as

$$\begin{aligned}\epsilon^2 &= \mathrm{E}\{(\hat{x}-x)^2 + (\hat{y}-y)^2\} \\ &\geq \text{trace}\left([\mathbf{J}_{\boldsymbol{\theta\theta}}^{-1}]_{2\times 2}\right),\end{aligned} \tag{19}$$

where trace$(\cdot)$ is the trace operator, and $[\mathbf{M}]_{2\times 2}$ is the first 2×2 block of $\mathbf{M}$.

Proposition 1: Given any unbiased estimates of x and y, denoted by $\hat{x}$ and $\hat{y}$, respectively. The variance of the error estimate is bounded by

$$\epsilon^2 \geq \frac{2\sum\limits_{b=M+1}^{B}\rho_b}{\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\sum\limits_{b_1=M+1}^{B}\sum\limits_{b_2=M+1}^{B}\rho_{b_1}\rho_{b_2}\sin^2(\phi_{b_2}-\phi_{b_1})}, \tag{20}$$

where ρ_b and ϕ_b are given by

$$\rho_b = 8\pi^2\frac{1}{c^2}\bar{\beta}^2 a_b^2 + \frac{1}{2}\kappa\frac{1}{d_0^2}\gamma_b^2\left(\frac{d_0}{d_b}\right)^{\gamma_b+2} \tag{21a}$$

$$\phi_b = \arctan\left(\frac{y-y_b}{x-x_b}\right). \tag{21b}$$

Proof: See Appendix B in [13]. ∎

Note that the derived result depends on only LoS portion. In another form, (21a) can be written as

$$\rho_b = \begin{cases} 8\pi^2\frac{1}{c^2}\bar{\beta}^2 a_b^2 & \text{; without path loss information,} \\ 8\pi^2\frac{1}{c^2}\bar{\beta}^2 a_b^2 + \frac{1}{2d_b^2}\gamma_b^2 a_b^2 & \text{; with path loss information.} \end{cases} \tag{22}$$

One can see that both assumptions are the same when $\gamma_b = 0$. However, the case of no path attenuation with $\gamma_b = 0$ does not exist in realistic environment. Therefore, the exploration of the path loss can reduce the CRB in the wireless NLoS geolocation.

In what follows, we investigate the improved position accuracy in the hybrid model. The hybrid model is referred to as the usual time delay model using the path attenuation parameterized in (6). The CRB in (20) can be concentrated on as

$$\epsilon^2 \geq \frac{32\pi^2 f^2 d_0^2\sum\limits_{b=M+1}^{B}\alpha_b}{c^2\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\sum\limits_{b_1=M+1}^{B}\sum\limits_{b_2=M+1}^{B}\alpha_{b_1}\alpha_{b_2}\sin^2(\phi_{b_2}-\phi_{b_1})}, \tag{23}$$

where α_b is given by

$$\alpha_b = \begin{cases} 8\pi^2\frac{1}{c^2}\bar{\beta}^2\left(\frac{d_0}{d_b}\right)^{\gamma_b} & \text{; Ordinary ToA,} \\ 8\pi^2\frac{1}{c^2}\bar{\beta}^2\left(\frac{d_0}{d_b}\right)^{\gamma_b} + \frac{1}{2d_0^2}\gamma_b^2\left(\frac{d_0}{d_b}\right)^{\gamma_b+2} & \text{; Hybrid SS-ToA.} \end{cases} \tag{24}$$

It is worth noting that the above result can be applied to a large class of signal, since we do not assume any structure of the transmitted signal $s(t)$. In [14], the estimation of position parameter is discussed by first estimating the time delay τ_b and then finding $\mathbf{p}$ from such a sufficient estimate. Herein, we assume that the the estimator design is beyond the scope of this work and it is curious to investigate the inherent accuracy limitation of the proposed hybrid method via the CRB.

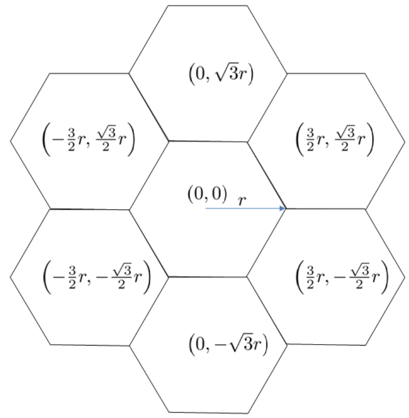

Fig. 1. Cellular system with cell radius r.

IV. Numerical Examples

The system is assumed to operate at a center frequency f_0 of 1.9 GHz [15] and an effective bandwidth $\bar{\beta}$ of $\frac{1}{\sqrt{3}}5$ MHz [14]. The transmitted signal-to-noise ratio, SNR, is defined by

$$\mathrm{SNR} = \frac{1}{\sigma_{\mathrm{n}}^2}\int_0^{T_{\mathrm{s}}}|s(t)|^2\mathrm{d}t = \frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}, \tag{25}$$

where T_{s} is the time period in which $s(t)$ is non-zero. For $d_0 = 100$ m, it is shown in [15] that $\kappa_{\mathrm{dB}} = 10\log_{10}(\kappa) \approx -78$ dB. Dividing (1) by σ_{n}^2, taking the logarithmic function of base 10, and multiplying both sides by 10, we obtain

$$10\log_{10}\left(\frac{E_b}{\sigma_{\mathrm{n}}^2}\right) = \kappa_{\mathrm{dB}} + 10\gamma_b\log_{10}\left(\frac{d_0}{d_b}\right) + \mathrm{SNR}_{\mathrm{dB}}, \tag{26}$$

where $\mathrm{SNR}_{\mathrm{dB}}$ is the transmitted SNR in dB defined by $\mathrm{SNR}_{\mathrm{dB}} = 10\log_{10}(\mathrm{SNR})$. In general, the term $10\log_{10}\left(\frac{E_b}{\sigma_{\mathrm{n}}^2}\right)$ can be considered as a received SNR. For a simple link budget, we assume $\gamma_b = 2$, $d_0 = 100$ m and $d_b = 1{,}000$ m. If the receiver desires to have the received SNR, $10\log_{10}\left(\frac{E_b}{\sigma_{\mathrm{n}}^2}\right)$, of 0 dB, the transmitter has to transmit a signal with $\mathrm{SNR}_{\mathrm{dB}} = 78.0168 - 20(2-3) = 98.0168$ dB. Taken into account the path attenuation, the transmitter should transmit a high SNR.

Consider a certain configuration of a cellular system. In seven hexagonal cells, let the origin of the Cartesian coordinate lie at the center of the central cell according to Fig. 1. The BSs are thus located at the center of each cell with

$$\mathbf{P} = r\begin{bmatrix} 0 & \frac{3}{2} & 0 & -\frac{3}{2} & -\frac{3}{2} & 0 & \frac{3}{2} \\ 0 & \frac{\sqrt{3}}{2} & \sqrt{3} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & -\sqrt{3} & -\frac{\sqrt{3}}{2} \end{bmatrix}^{\mathrm{T}}, \tag{27}$$

where r is the cell radius. The mobile station is located at $\mathbf{p} = \frac{1}{2}r\cos\left(\frac{1}{6}\pi\right)\left[\cos\left(\frac{1}{6}\pi\right) \quad \sin\left(\frac{1}{6}\pi\right)\right]^{\mathrm{T}}$ m. In what follows, we assume that the cell radius can be varied. The mobile position is $\frac{\sqrt{3}}{4}r$ m apart from the center of the central cell. With respect

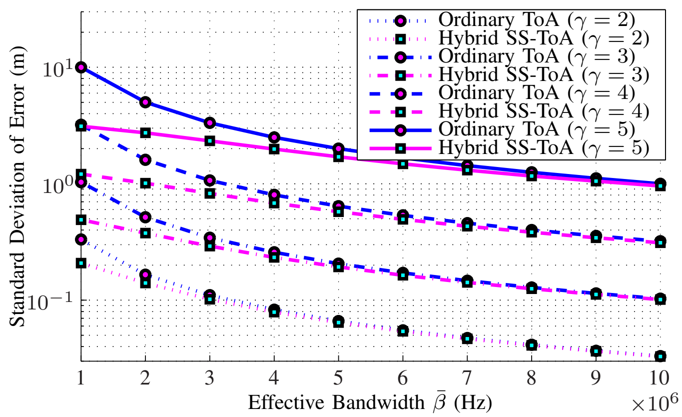

Fig. 2. Cramér-Rao lower bound as a function of the effective bandwidth for several path loss exponents with $\mathrm{SNR}_{\mathrm{dB}} = 110$ dB, B (number of BSs) $= 7$, M (number of BSs receiving NLoS signals) $= 3$, d_0 (close-in distance) $= 4$ m and r (cell radius) $= 20$ m.

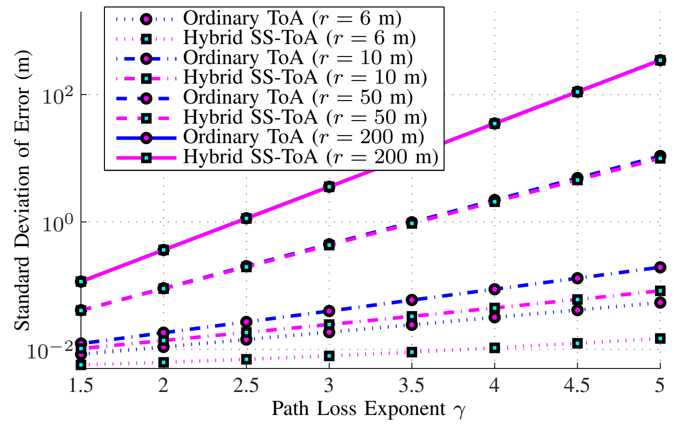

Fig. 3. Cramér-Rao lower bound as a function of the path loss exponent for several cell radii with $\mathrm{SNR}_{\mathrm{dB}} = 120$ dB, $\bar{\beta}$ (effective bandwidth) $= \frac{1}{\sqrt{3}}5$ MHz, B (number of BSs) $= 7$, M (number of BSs receiving NLoS signals) $= 3$ and d_0 (close-in distance) $= 4$ m.

to the MS position, the associated angles of BSs become

$$\boldsymbol{\phi} = \begin{bmatrix} \arctan\left(\frac{-\frac{\sqrt{3}}{8}r}{-\frac{3}{8}r}\right) \\ \arctan\left(\frac{\frac{\sqrt{3}}{2}r-\frac{\sqrt{3}}{8}r}{\frac{3}{2}r-\frac{3}{8}r}\right) \\ \arctan\left(\frac{\sqrt{3}r-\frac{\sqrt{3}}{8}r}{-\frac{3}{8}r}\right) \\ \arctan\left(\frac{\frac{\sqrt{3}}{2}r-\frac{\sqrt{3}}{8}r}{-\frac{3}{2}r-\frac{3}{8}r}\right) \\ \arctan\left(\frac{-\frac{\sqrt{3}}{2}r-\frac{\sqrt{3}}{8}r}{-\frac{3}{2}r-\frac{3}{8}r}\right) \\ \arctan\left(\frac{-\sqrt{3}r-\frac{\sqrt{3}}{8}r}{-\frac{3}{8}r}\right) \\ \arctan\left(\frac{-\frac{\sqrt{3}}{2}r-\frac{\sqrt{3}}{8}r}{\frac{3}{2}r-\frac{3}{8}r}\right) \end{bmatrix} = \begin{bmatrix} -150.0000^\circ \\ 30.0000^\circ \\ 103.8979^\circ \\ 160.8934^\circ \\ -150.0000^\circ \\ -100.8934^\circ \\ -43.8979^\circ \end{bmatrix}. \quad (28)$$

We assume that the first M base stations receive the NLoS signals. The positioning accuracy is calculated from the square root of (23).

In Fig. 2, the CRB is shown as a function of the effective bandwidth $\bar{\beta}$. We can see the effect of path attenuation in NLoS geolocation. When the path loss exponent is increased, the inherent error of mobile position is higher. The CRB of hybrid wireless geolocation is lower than that of the ordinary ToA method, especially for low effective bandwidth of the transmitted signal. In the hybrid approach, we have $\alpha_b = 8\pi^2 \frac{1}{c^2}\bar{\beta}^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b} + \frac{1}{2d_0^2}\gamma_b^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b+2}$. One can see that as $\bar{\beta}$ increases, the second term, which is the contribution from signal strength, becomes relatively minor.

In Fig. 3, the CRB is investigated as a function of the path loss exponent for several cell radii. The close-in distance $d_0 = 4$ m is chosen for indoor scenario. We can see that the smaller the cell radius, the better the positioning accuracy. This is because the received power decreases with the increase in the distance between MS and BS. When the cell is more stretched out, the SNR is reduced. Let us reconsider $\alpha_b = 8\pi^2 \frac{1}{c^2}\bar{\beta}^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b} + \frac{1}{2d_0^2}\gamma_b^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b+2}$. Since the ratio $\frac{d_0}{d_b}$ is always less than one due to the far field assumption, the performance improvement by $\gamma_b^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b+2}$ will be significant in the case of small d_b, i.e. small cell, and high path loss exponent.

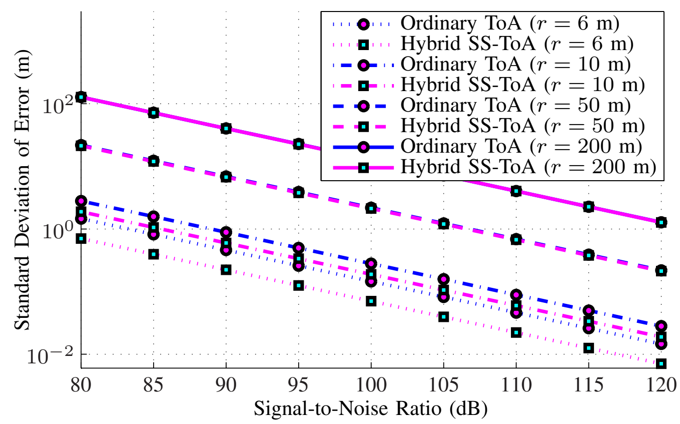

Fig. 4. Cramér-Rao lower bound as a function of the signal-to-noise ratio for several cell radii with $\bar{\beta}$ (effective bandwidth) $= \frac{1}{\sqrt{3}}5$ GHz, B (number of BSs) $= 7$, M (number of BSs receiving NLoS signals) $= 3$, γ (path loss exponent) $= 2.55$ and d_0 (close-in distance) $= 4$ m.

For the office building picocells on the same floor in Fig. 4, the path loss exponent γ and the close-in distance d_0 are assumed to be 2.55 and 4 m, respectively. This configuration is in accordance with the outdoor scenario [4, p. 47], [5, p. 109] and [15]. In Fig. 4, the CRB is observed as a function of SNR for several cell radii. It can be seen that the positioning accuracy is better for higher SNR. The performance improvement is more obvious in a smaller cell size.

In Fig. 5, the CRB is shown as a function of the cell radius for several SNRs. When the cell radius is large, the performance improvement of the hybrid approach is gradually minor.

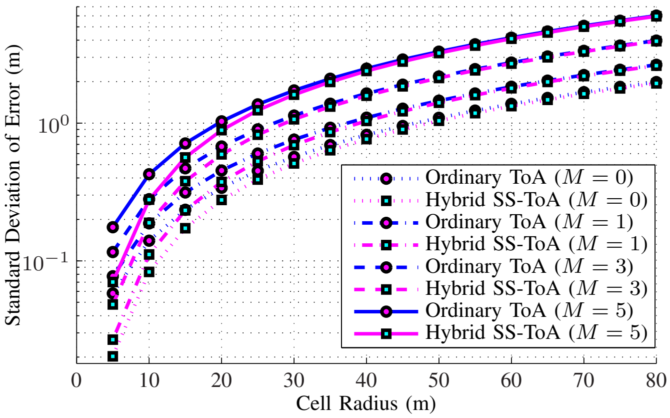

Fig. 5. Cramér-Rao lower bound as a function of cell radius for several numbers of NLoS base staions with $\mathrm{SNR}_{\mathrm{dB}}$ (signal-to-noise ratio) $= 100$ dB, B (number of BSs) $= 7$, $\gamma = 2.55$ and d_0 (close-in distance) $= 4$ m.

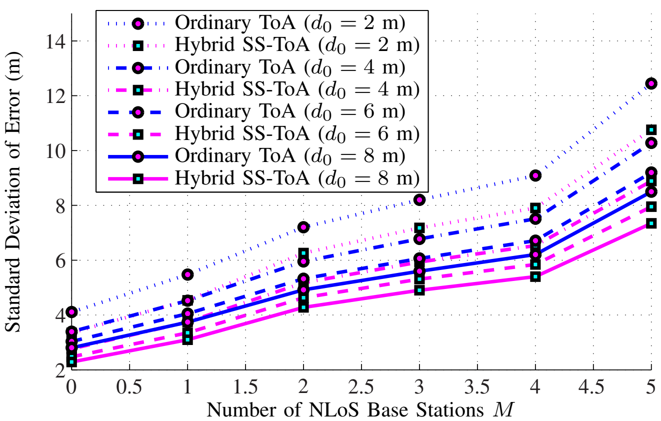

Fig. 6. Cramér-Rao lower bound as a function of the number of NLoS base stations for several close-in distances with $\bar{\beta}$ (effective bandwidth) $= \frac{1}{\sqrt{3}}5$ MHz, B (number of BSs) $= 7$, $\mathrm{SNR}_{\mathrm{dB}}$ (signal-to-noise ratio) $= 80$ dB, and r (cell radius) $= 20$ m.

In Fig. 6, the bound of positioning error is illustrated as a function of the number of NLoS BSs for several close-in distances. When there is fewer information of the LoS signals, i.e. smaller number of LoS BSs, the inherent accuracy is worse. In addition, the larger the close-in distance, the lower the error variance. It can be seen that the close-in distance has a significant impact to the inherent accuracy of the mobile position.

V. Conclusion

In this paper, we have presented an approach for hybrid wireless geolocation using time delay in the presence of NLoS propagation by considering additional path loss information. Since the signal energy is attenuated during the propagation, the transmitted SNR needs to be high at the transmitter. The Cramér-Rao lower bound is derived from the proposed method. This expression generalizes the recent result in [1] by exploring an additional term $\frac{1}{2d_0^2}\gamma_b^2\left(\frac{d_0}{d_b}\right)^{\gamma_b+2}$ in (24). The obtained result indicates that the path attenuation contains the information of the path loss exponent and the distance between the mobile and base station. Numerical results illustrate that when the path loss is included into the wireless geolocation, the estimation accuracy in small cells, e.g. office building picocells, is improved comparing with the ordinary ToA method. The performance improvement is significant, especially in the system that 1) invokes the signal with a small effective bandwidth, 2) has a small distance between the mobile and base stations, and 3) has a high value of the path loss exponent, i.e. severe path attenuation. For future work, the analysis and the design of a time delay estimator remains an open problem for the hybrid SS-ToA approach.

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11:10 h - 12:50 h, Tuesday, April 27, 2009, Floor 1 Foyer 1, Multiple antenna systems and space-time processing posters I, The 69th IEEE Vehicular Technology Conference, Hilton Diagonal Mar, Barcelona, Spain

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A Hybrid SS-ToA Wireless NLoS Geolocation Based on Path Attenuation: Cramér-Rao Bound

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Received Energy

The received energy at the b-th BS can be expressed by

$$E_b = \frac{d_0^{\gamma_b}}{d_b^{\gamma_b}}\kappa E_s, \quad (1)$$

where

- d_0 is the close-in reference in the far field region,
- d_b is the distance between the MS and the b-th BS,
- γ_b is the path loss exponent at the b-th BS,
- $E_s = \int_{-\infty}^{\infty}|s(t)|^2 \mathrm{d}t$ is the energy of a transmitted signal $s(t)$,
- and κ is the constant depending on antenna characteristics and average channel attenuation given by $\kappa = \frac{c^2}{16\pi^2 f_0^2 d_0^2}$, with the center frequency f_0, and the speed of light c.

Path Gain

Since $E_b = a_b^2 E_s$, the unitless amplitude is given by

$$a_b = \frac{1}{\left(\sqrt{\left(\frac{\tilde{x}_b}{d_0}\right)^2 + \left(\frac{\tilde{y}_b}{d_0}\right)^2} + \frac{l_b}{d_0}\right)^{\frac{1}{2}\gamma_b}}\sqrt{\kappa}. \quad (2)$$

Time of Arrival

Let τ_b be the time delay of received signal at the b-th BS, i.e.

$$\tau_b(x, y, l_b) = \frac{1}{c}\left(\sqrt{\tilde{x}_b^2 + \tilde{y}_b^2} + l_b\right), \quad (3)$$

where $\tilde{x}_b = x - x_b$, $\tilde{y}_b = y - y_b$, and $l_b = 0$; for $b \in \{M+1, M+2, \ldots, B\}$.

Received Signal

The received baseband signal can be written as

$$r_b(t) = a_b s(t - \tau_b) + n_b(t), \quad (4)$$

where $s(t)$ is the known waveform, and $n_b(t)$ is an additive noise at the b-th BS and assumed to be a complex-valued white Gaussian process with zero mean and variance σ_n^2.

Cramér-Rao Lower Bound

The CRB can be written as

$$\epsilon^2 = \mathrm{E}\{(\hat{x}-x)^2 + (\hat{y}-y)^2\} \geq \frac{32\pi^2 f_0^2 d_0^2 \sum_{b=M+1}^{B} \alpha_b}{\frac{E_s}{\sigma_n^2} c^2 \sum_{b_1=M+1}^{B}\sum_{b_2=M+1}^{B} \alpha_{b_1}\alpha_{b_2}\sin^2(\phi_{b_2} - \phi_{b_1})}, \quad (5)$$

where α_b is given by

$$\alpha_b = \begin{cases} \frac{1}{c^2} 8\pi^2 \bar{\beta}^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b}, & \text{Ordinary ToA}, \\ \frac{1}{c^2} 8\pi^2 \bar{\beta}^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b} + \frac{1}{2d_0^2}\gamma_b^2 \left(\frac{d_0}{d_b}\right)^{\gamma_b+2}, & \text{Hybrid SS-ToA}, \end{cases} \quad (6)$$

and ϕ_b is given by $\phi_b = \arctan\left(\frac{y - y_b}{x - x_b}\right)$.

Numerical Examples: Cellular System

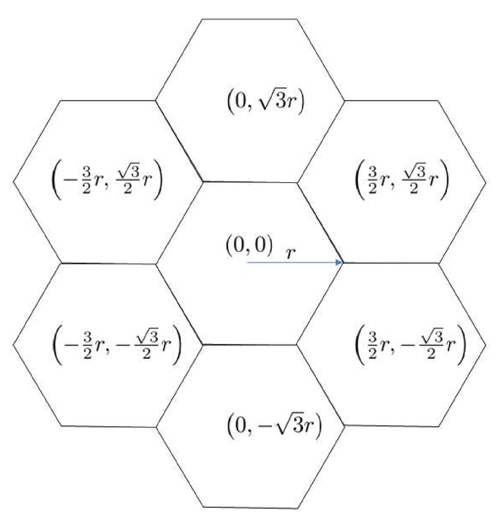

Fig. 1: Cellular system with cell radius r.

Numerical Examples: CRB

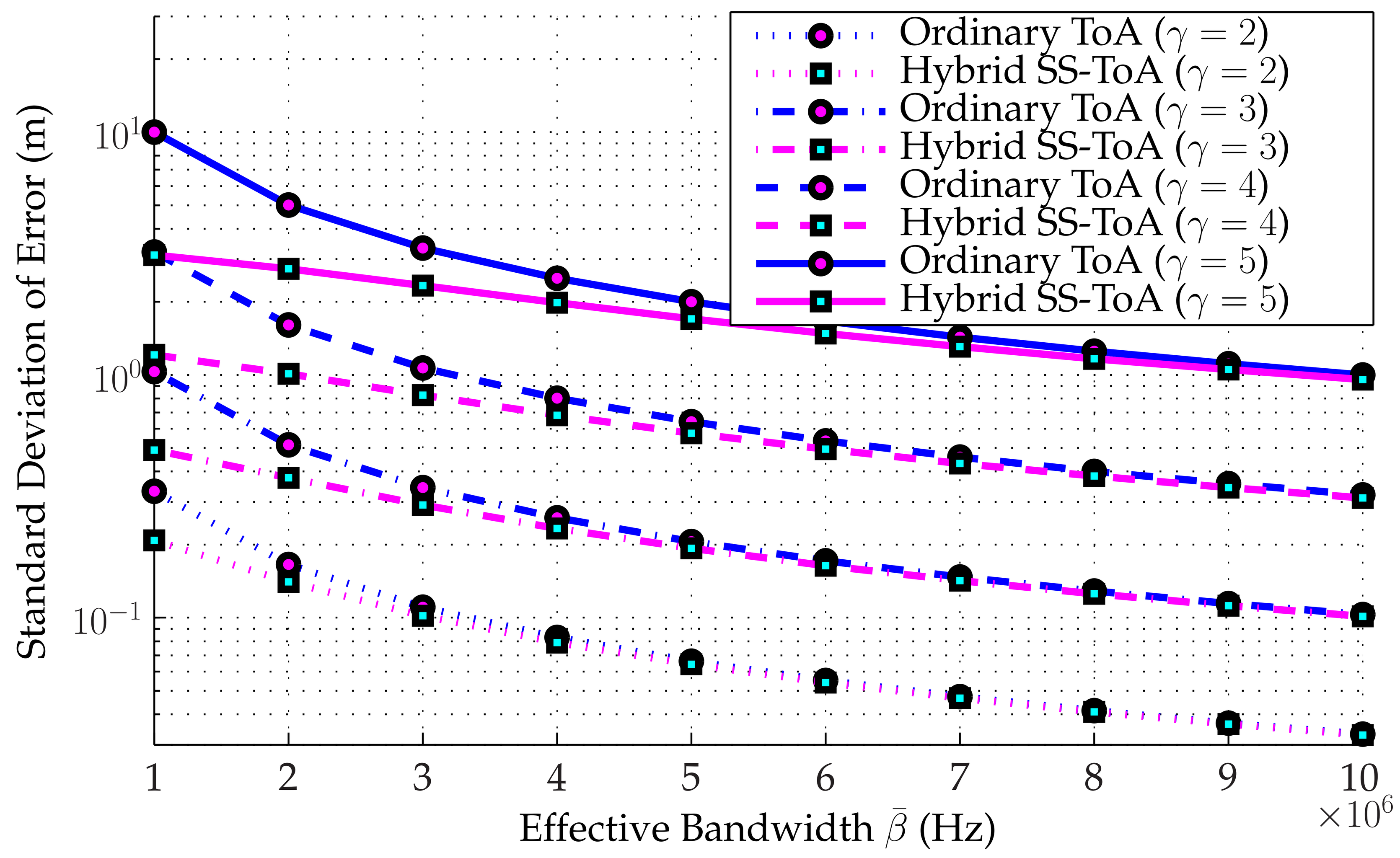

Fig. 2: Cramér-Rao lower bound as a function of the effective bandwidth for several path loss exponents with $\mathrm{SNR_{dB}} = 110$ dB, the number of the BSs $B = 7$, the number of the BSs receiving NLoS signals $M = 3$, the close-in distance $d_0 = 4$ m and the cell radius $r = 20$ m.

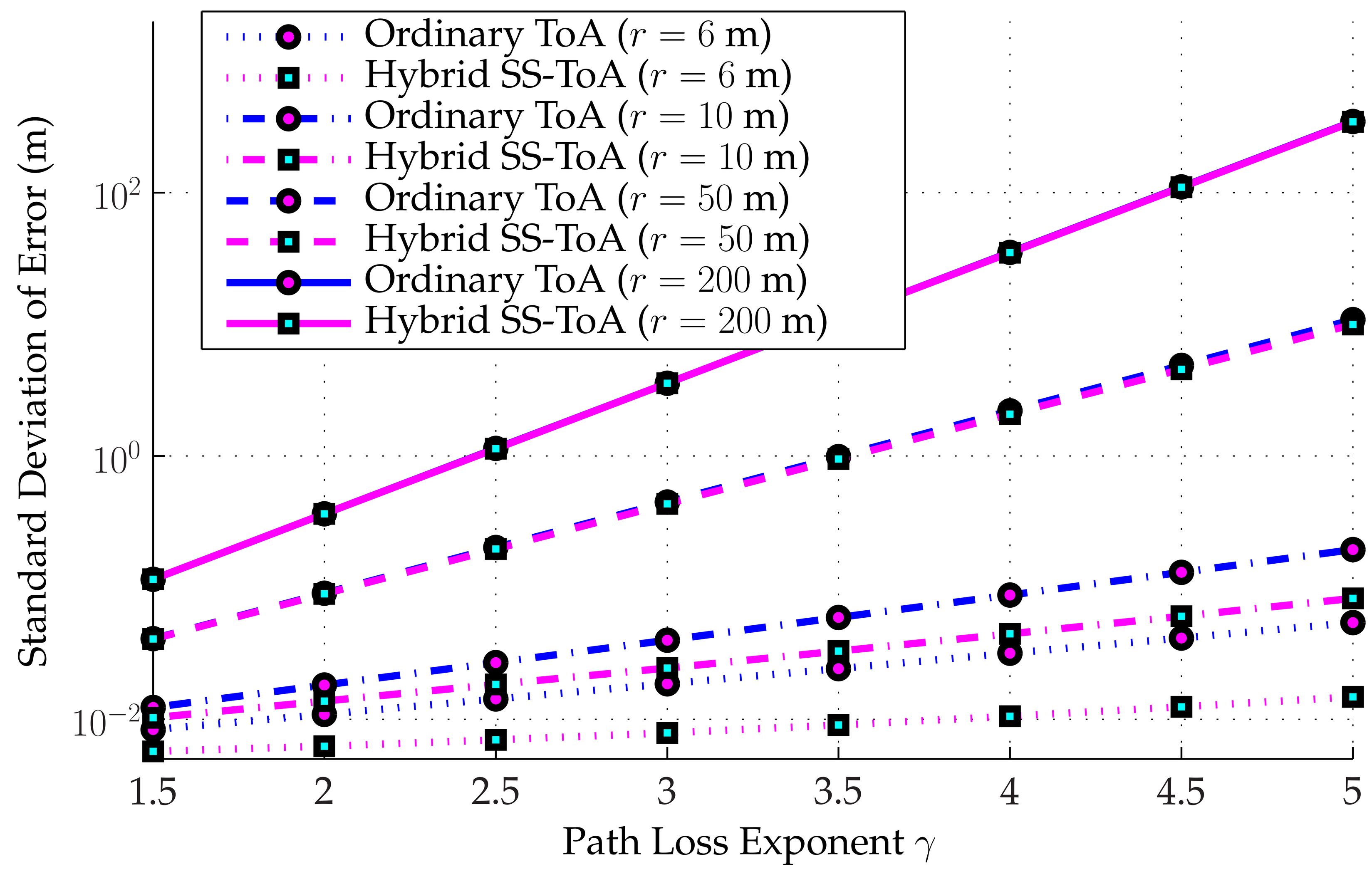

Fig. 3: Cramér-Rao lower bound as a function of the path loss exponent for several cell radii with $\mathrm{SNR_{dB}} = 120$ dB, the effective bandwidth $\bar{\beta} = \frac{1}{\sqrt{3}}5$ MHz, the number of the BSs $B = 7$, the number of the BSs receiving NLoS signals $M = 3$ and the close-in distance $d_0 = 4$ m.

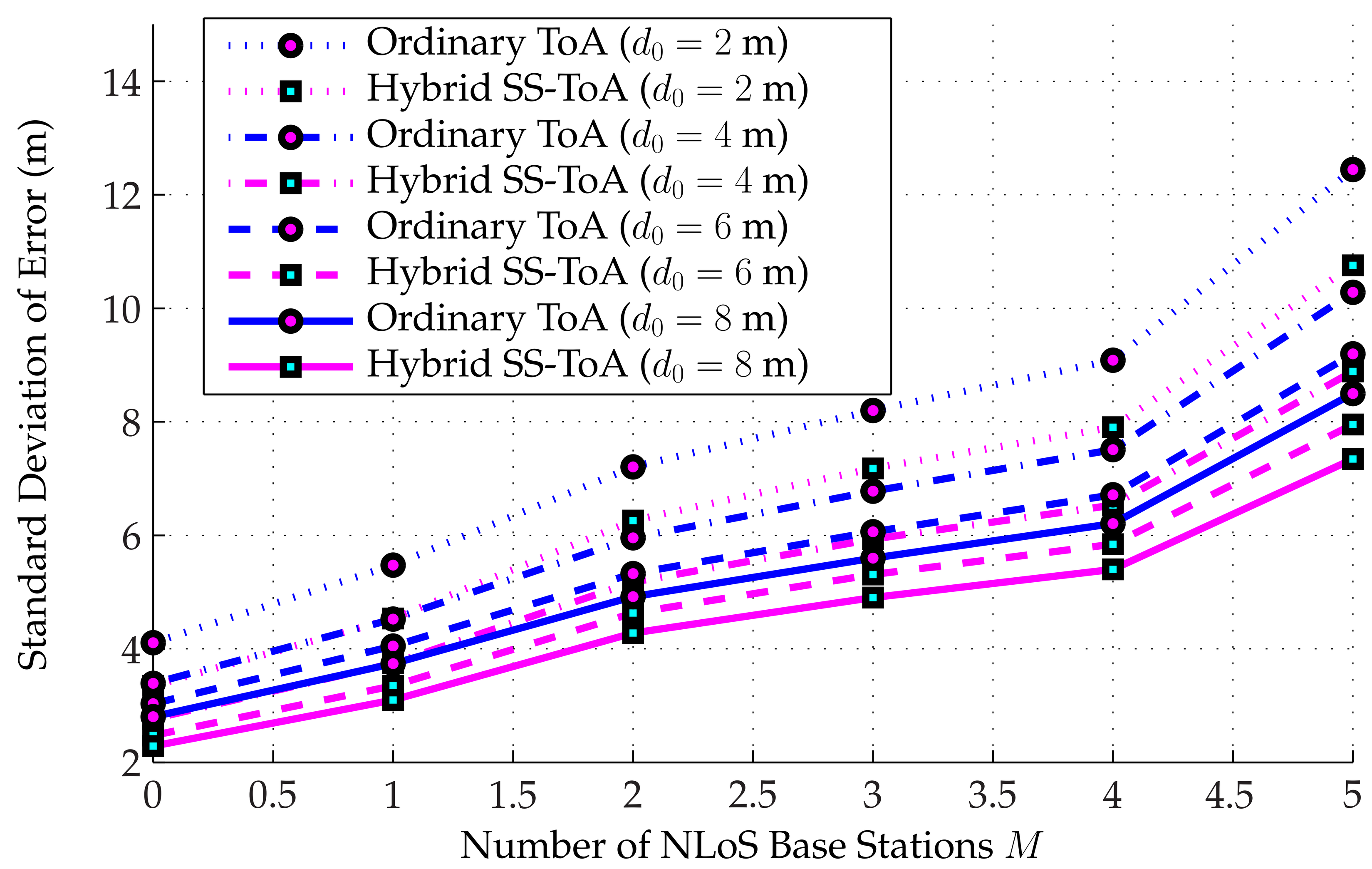

Fig. 4: Cramér-Rao lower bound as a function of the number of the NLoS base stations for several close-in distances with the effective bandwidth $\bar{\beta} = \frac{1}{\sqrt{3}}5$ MHz, the number of the BSs $B = 7$, the signal-to-noise ratio $\mathrm{SNR_{dB}} = 80$ dB, and the cell radius $r = 20$ m.