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TIME-OF-ARRIVAL ESTIMATION IN PATH ATTENUATION

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ABSTRACT

We consider a time-of-arrival (ToA) estimation in the presence of path attenuation. Maximum correlation (MC) is revisited and maximum likelihood (ML) is newly derived to estimate the ToA. It reveals that for low effective bandwidth, short distance and large path loss exponent, the ML has a smaller error variance than the MC. Numerical examples illustrate that the ML outperforms the MC.

***Index Terms*—** Time of arrival estimation, Maximum likelihood estimation.

1. INTRODUCTION

The problem of estimating some waveform parameters in additive Gaussian noise is investigated for long time ago [1]. Extensive literatures indicate that time delay estimation plays an important role in applied signal processing, which is intimately related to detection, synchronization, array processing, surveillance, range finding, tracking and geolocation [2]. Time-of-arrival (ToA) estimation is deemed a favorable technique for those applications, since its accuracy depends on the signal bandwidth, which allows the designer to adjust the signal according to a desired precision. In classical time delay problem, the path gain describes the propagation effect [3, 4, 5]. In the previous works, the path gain is treated to be distance-independent and hence maximum likelihood (ML) solution yields a maximum correlation (MC) between the received signal and a delayed replica of the transmitted signal. Recently, the vast development of channel modeling reveals that signal energy is attenuated in the channel [6, 7, 8], thus calling for the exploration of path loss in the ToA estimation.

In this paper, we consider the ToA estimation in which the path gain is distance-dependent. The ToA estimation in this manner can be applied to any localization application, which uses the ToA as the key feature of determining the target position. To estimate the ToA, the traditional MC is applied and can be regarded as the ToA estimation without the information of path loss.

Contributions of the paper are to investigate the benefit of deploying the attenuation of the path gain. We derive the ML estimator and its asymptotic error performance for the ToA estimation. It is worthwhile to note that the new ML is different from the MC in that it includes the knowledge of path loss, thus constituting a hybrid signal strength (SS)-ToA approach. Compared to the MC, the ML allows the investigation of the path attenuation deployment. The explicit form of the error performance of both estimators is derived using a standard and unified framework based on expanding the Taylor series of the objective function. The analytic results point out that the ML has a smaller error variance than the MC. As a consequence, it can be implied that the exploitation of the path loss increases the accuracy of time delay estimation. Numerical results illustrate that the ML well outperforms the MC for a low effective bandwidth of the transmitted signal, a small distance between the transmitter and receiver, and a large path loss exponent.

2. TRANSCEIVER MODEL

The received energy at the receiver can be expressed by (see e.g.[7, p. 38] and [8, p. 46])

$$E = \kappa \frac{d_0^{\gamma}}{d^{\gamma}} E_{\mathrm{s}}, \tag{1}$$

where d_0 is the close-in reference in the far field region, d is the distance between the receiver and the transmitter, γ is the path loss exponent, $E_{\mathrm{s}} = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ is the energy of transmitted signal $s(t)$, and κ is the unitless constant depending on antenna characteristics and average channel attenuation given by $\kappa = \frac{c^2}{16\pi^2 f_0^2 d_0^2}$, with the center frequency f_0 and the speed of light c. The received baseband signal is

$$r(t) = a_0 s(t - \tau_0) + n(t), \tag{2}$$

where $s(t)$ is a known waveform, a_0 and τ_0 are the amplitude and the time delay of propagation to the receiver, respectively, and $n(t)$ is an additive noise at the receiver and assumed to be a complex-valued zero-mean white Gaussian process with a variance of σ_{n}^2. The channel is assumed herein to be static in such a way that the unknown parameter τ_0 is invariant over the observation period $t \in (0, T]$ and the large-scale fading is considered as a spatial average over the small-scale fluctuations of the signals [9, p. 847]. Assuming $E = a^2 E_{\mathrm{s}}$, the path gain is given by

$$a_0 = \sqrt{\kappa}\left(\frac{d_0}{c\tau_0}\right)^{\frac{1}{2}\gamma}. \tag{3}$$

Let the solution of the homogeneous Fredholm integral equation $\int_0^T \varphi(t,\acute{t})f_k(\acute{t})\mathrm{d}\acute{t} = \lambda_k f_k(t)$ for $k \in \{1,2,\ldots,K\}$ be the eigenvalue λ_k and the orthonormal function $f_k(t)$, where the kernel $\varphi(t,\acute{t})$ is the eigenfunction, which is the noise autocovariance function. Using the Karhunen-Loève (KL) expansion (see e.g. [4, p. 37], and [10, p. 298]), the signal can be sampled from $f_k(t)$ according to

$$r(t) = \lim_{K\to\infty}\sum_{k=1}^{K} r_k f_k(t), \tag{4}$$

where the received signal sample is given by $r_k = \int_0^T f_k(t) r(t)\mathrm{d}t$. From (2), the received signal sample can be expressed by

$$r_k = a_0 s_k + n_k, \tag{5}$$

where the signal and noise samples are given by $s_k = \int_0^T f_k(t)s(t-\tau_0)\mathrm{d}t$ and $n_k = \int_0^T f_k(t)n(t)\mathrm{d}t$. Assume that the basis function $f_k(t)$ is chosen such that the noise samples $\{n_k\}_{k=1}^K$ are identically and independently distributed. The probability density function (PDF) of the complex Gaussian random multivariate $\{r_k\}_{k=1}^K$ can be written as

$$p(r_1,\ldots,r_K|\tau_0) = \frac{1}{(\pi\sigma_\mathrm{n}^2)^K}\mathrm{e}^{-\frac{1}{\sigma_\mathrm{n}^2}\sum_{k=1}^{K}|r_k - a_0 s_k|^2}. \tag{6}$$

Given the continuous signal $r(t);\, t\in(0,T]$, the likelihood of τ_0 can be written in logarithm scale as

$$\begin{aligned} l(\tau_0|r(t);t\in(0,T]) &= \lim_{K\to\infty}\ln\left(p(r_1,\ldots,r_K|\tau_0)\right) \\ &\doteq -\frac{1}{\sigma_\mathrm{n}^2}\int_0^T |r(t)-a_0 s(t-\tau_0)|^2\mathrm{d}t, \end{aligned} \tag{7}$$

where $\doteq$ is the equivalence by neglecting an irrelevant term.

3. TOA ESTIMATION AND ERROR PERFORMANCE

In this section, we consider the theoretical error performance of a desired ToA estimator based on a cost function. Let $g(\tau)$ be an objective function, which is continuous and differentiable up to the second order. Taking the first-order Taylor series of the derivative $\frac{\partial}{\partial\tau}g(\tau)$ around the true value τ_0, we arrive at (see e.g. [3, eq. (17-9.2)] and [4, eq. (6-50)])

$$\frac{\partial}{\partial\tau}g(\tau) = \frac{\partial}{\partial\tau}g(\tau)\Big|_{\tau=\tau_0} + \frac{\partial^2}{\partial\tau^2}g(\tau)\Big|_{\tau=\tau_0}(\tau-\tau_0)+o((\tau-\tau_0)^2), \tag{8}$$

where the little oh of $u(\tau-\tau_0) = o(v(\tau-\tau_0))$ stands for $\lim_{\tau\to\tau_0}\frac{u(\tau-\tau_0)}{v(\tau-\tau_0)} = 0$. At the estimated point $\tau = \hat{\tau}$, we obtain $0 = \frac{\partial}{\partial\tau}g(\tau)\big|_{\tau=\tau_0} + (\hat{\tau}-\tau_0)\,\frac{\partial^2}{\partial\tau^2}g(\tau)\big|_{\tau=\breve{\tau}}$ (see e.g. [11, p. 240]), where $\breve{\tau}$ lies in the line segment between τ_0 and $\hat{\tau}$. For a continuous derivative $\frac{\partial^2}{\partial\tau^2}g(\tau)$, the quantity $\frac{\partial^2}{\partial\tau^2}g(\tau)\big|_{\tau=\breve{\tau}}$ converges to $\mathrm{E}_{n(t)}\left\{\frac{\partial^2}{\partial\tau^2}g(\tau)\big|_{\tau=\tau_0}\right\}$ with probability one. As a result, the time delay estimation error converges to

$$\hat{\tau}-\tau_0 \cong -\frac{\frac{\partial}{\partial\tau}g(\tau)\big|_{\tau=\tau_0}}{\mathrm{E}_{n(t)}\left\{\frac{\partial^2}{\partial\tau^2}g(\tau)\big|_{\tau=\tau_0}\right\}}, \tag{9}$$

where $\cong$ is the equality by neglecting $o((\tau-\tau_0)^2)$.

3.1. Maximum Correlation

The maximum correlation lies in the same idea as the matched filter in binary information detection. It corresponds to the maximum likelihood for the case in which the path gain is independent of the ToA. We shall refer the MC to as the classical method for the ToA estimation. Let $\rho(\tau)$ be a correlation function between the received signal and a delayed replica of the transmitted waveform, i.e.

$$\rho(\tau) = \int_0^T \Re\left(r^*(t)s(t-\tau)\right)\mathrm{d}t, \tag{10}$$

where $\Re(\cdot)$ is the real part and $(\cdot)^*$ is the complex conjugate. The solution of the MC is given by $\hat{\tau}_\mathrm{MC} = \arg\max_\tau \rho(\tau)$. Following from (9), the time delay error of the MC is

$$\hat{\tau}_\mathrm{MC}-\tau_0 \cong \frac{\dot{\rho}_\mathrm{ns}(\tau_0)}{4\pi^2 E_\mathrm{s}\bar{\beta}^2 a_0}, \tag{11}$$

where $\dot{\rho}_\mathrm{ns}(\tau) = \int_0^T \Re\left(n^*(t)\frac{\partial}{\partial\tau}s(t-\tau)\right)\mathrm{d}t$. Using $\mathrm{E}_{n(t)}\{\dot{\rho}_\mathrm{ns}(\tau_0)\} = 0$ and $\mathrm{E}_{n(t)}\{\dot{\rho}_\mathrm{ns}^2(\tau_0)\} = 2\pi^2\bar{\beta}^2 E_\mathrm{s}\sigma_\mathrm{n}^2$, the bias and error variance of the MC estimate are written as

$$\mathrm{E}_{n(t)}\{\hat{\tau}_\mathrm{MC}-\tau_0\} = 0, \tag{12a}$$

$$\mathrm{E}_{n(t)}\{(\hat{\tau}_\mathrm{MC}-\tau_0)^2\} = \frac{1}{\frac{E_\mathrm{s}}{\sigma_\mathrm{n}^2}8\pi^2\bar{\beta}^2 a_0^2}, \tag{12b}$$

where $\bar{\beta}$ is the effective (root-mean-square) bandwidth defined by

$$\bar{\beta} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2|S(f)|^2\mathrm{d}f}{\int_{-\infty}^{\infty}|S(f)|^2\mathrm{d}f}}, \tag{13}$$

with $S(f)$ being the Fouriér transform of $s(t)$. The error performance shown above is equivalent to the standard benchmark, i.e. the Cramér-Rao bound (CRB) for the time delay estimation based on the distance-independent path gain [3, 4, 5].

3.2. Maximum Likelihood

The maximum likelihood is derived from the likelihood function in (7). It is a classical and optimal method in the sense of asymptotic error variance for the probabilistic model [12]. Let $\zeta(\tau)$ be the ML objective function defined by

$$\zeta(\tau) = a^2(\tau)E_{\rm s} - 2a(\tau)\rho(\tau), \tag{14}$$

where $a(\tau) = \sqrt{\kappa}\left(\frac{d_0}{c\tau}\right)^{\frac{1}{2}\gamma}$. The ML estimate can be given by $\hat{\tau}_{\rm ML} = \arg\min_{\tau} \zeta(\tau)$. The ML time delay estimation error converges to

$$\hat{\tau}_{\rm ML} - \tau_0 \cong \frac{\tau_0(2\tau_0\dot{\rho}_{\rm ns}(\tau_0) - \gamma\rho_{\rm ns}(\tau_0))}{E_{\rm s}(\frac{1}{2}\gamma^2 + 8\pi^2\bar{\beta}^2\tau_0^2)a_0}, \tag{15}$$

where $\rho_{\rm ns}(\tau)) = \int_0^T \Re\left(n^*(t)s(t-\tau)\right){\rm d}t$. Using $\mathrm{E}_{n(t)}\{\rho_{\rm ns}(\tau_0)\} = 0$, $\mathrm{E}_{n(t)}\{\dot{\rho}_{\rm ns}(\tau_0)\} = 0$, $\mathrm{E}_{n(t)}\{\rho^2_{\rm ns}(\tau_0)\} = \frac{1}{2}E_{\rm s}\sigma_{\rm n}^2$, $\mathrm{E}_{n(t)}\{\dot{\rho}^2_{\rm ns}(\tau_0)\} = 2\pi^2\bar{\beta}^2E_{\rm s}\sigma_{\rm n}^2$ and $\mathrm{E}_{n(t)}\{\rho_{\rm ns}(\tau_0)\dot{\rho}_{\rm ns}(\tau_0)\} = 0$, the bias and error variance remain

$$\mathrm{E}_{n(t)}\{\hat{\tau}_{\rm ML} - \tau_0\} = 0, \tag{16a}$$

$$\mathrm{E}_{n(t)}\{(\hat{\tau}_{\rm ML} - \tau_0)^2\} = \frac{1}{\frac{E_{\rm s}}{\sigma_{\rm n}^2}a_0^2\left(8\pi^2\bar{\beta}^2 + \frac{1}{2\tau_0^2}\gamma^2\right)}. \tag{16b}$$

It is straightforward to verify that the CRB yields the same expression as (16b). We can see that the ratio between the ML and the MC error variances is given by

$$\frac{\mathrm{E}_{n(t)}\{(\hat{\tau}_{\rm ML} - \tau_0)^2\}}{\mathrm{E}_{n(t)}\{(\hat{\tau}_{\rm MC} - \tau_0)^2\}} = \frac{1}{1 + \frac{1}{16\pi^2\bar{\beta}^2\tau_0^2}\gamma^2}, \tag{17}$$

which indicates that the ML error variance in (16b) is less than that of the MC in (12b). The ML error variance becomes the MC error variance for i) large distance, ii) large effective bandwidth and iii) $\gamma = 0$. The latter condition is however irrelevant, since the case $\gamma = 0$ means no path attenuation presents. In fact, for free space the path loss exponent is of $\gamma = 2$ (see e.g. [13, p. 88]), while for wireless environment, the path loss exponent can be less than 2 but larger than 0 [8, p. 47]. The performance improvement thus exists and is significant, when τ_0 and $\bar{\beta}$ are small and γ is large.

4. NUMERICAL RESULTS

The ToA estimation in the path attenuation developed above can be applied to any analytic signal. In this work, we consider the orthogonal frequency division multiplexing (OFDM) signal.

4.1. OFDM Signal

The OFDM signal of the duration $t \in [0, T_s]$ is given by (see e.g. [14])

$$\tilde{s}(t) = \sum_{k=0}^{N-1} b_k \mathrm{e}^{j2\pi f_k t}, \tag{18}$$

where $\{b_k\}_{k=0}^{N-1}$ is the block of N complex data symbols chosen from a signal constellation, such as quadrature amplitude modulation (QAM) or phase shift keying (PSK), and $f_k = f_0 + \frac{1}{T_s}k$. At the receiver, the OFDM signal is down-converted to baseband representation, i.e. $s(t) = \tilde{s}(t)\mathrm{e}^{-j2\pi f_0 t}$. For $s(t);\ t \in [0, T_s]$, the effective bandwidth can be written as

$$\bar{\beta} = \frac{1}{T_s}\sqrt{\frac{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} k_1k_2b_{k_1}b^*_{k_2}\mathrm{sinc}\left(\pi(k_1-k_2)\right)\mathrm{e}^{j\pi(k_1-k_2)}}{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} b_{k_1}b^*_{k_2}\mathrm{sinc}\left(\pi(k_1-k_2)\right)\mathrm{e}^{j\pi(k_1-k_2)}}}, \tag{19}$$

where $\mathrm{sinc}(\phi) = \frac{1}{\phi}\sin(\phi)$ is the unnormalized sine cardinal function. We found that the effective bandwidth in (19) provides the same value as that of the following.

Lemma 1 (Effective bandwidth of baseband OFDM signal)**.** *According to the baseband representation in* (2)*, the effective bandwidth of the baseband OFDM signal is given by*[1]

$$\bar{\beta} = \frac{1}{T_s}\sqrt{\frac{1}{6}(2N^2 - 3N + 1)}. \tag{20}$$

Proof. Using the Parseval's theorem, the signal energy is $\int_{-\infty}^{\infty}|\tilde{S}(f)|^2{\rm d}f = \int_{-\infty}^{\infty}|\tilde{s}(t)|^2{\rm d}t = \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} b_{k_1}b^*_{k_2}\lim_{T_\infty\to\infty}2T_\infty\,\mathrm{sinc}\left(\frac{1}{T_s}2\pi(k_1-k_2)T_\infty\right)$. From the derivative property of the Fouriér transform, the second moment of signal frequency reads $\int_{-\infty}^{\infty} f^2|\tilde{S}(f)|^2{\rm d}f = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\left|\frac{\partial}{\partial t}\tilde{s}(t)\right|^2{\rm d}t = \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} b_{k_1}b^*_{k_2}f_{k_1}f_{k_2}\lim_{T_\infty\to\infty}2T_\infty\mathrm{sinc}\left(\frac{1}{T_s}2\pi(k_1-k_2)T_\infty\right)$. The square of the effective bandwidth can be written as $\bar{\beta}^2 = \lim_{T_\infty\to\infty}\frac{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} f_{k_1}f_{k_2}b_{k_1}b^*_{k_2}\mathrm{sinc}\left(\frac{1}{T_s}2\pi(k_1-k_2)T_\infty\right)}{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} b_{k_1}b^*_{k_2}\mathrm{sinc}\left(\frac{1}{T_s}2\pi(k_1-k_2)T_\infty\right)}$. Using $\mathrm{sinc}(\infty) = 0$ and $|b_k|^2 = |b_{\acute{k}}|^2;\ k \neq \acute{k}$, we have $\bar{\beta} = \sqrt{f_0^2 + \frac{1}{T_s}(N-1)\left(f_0 + \frac{1}{6T_s}(2N-1)\right)}$, which reduces to (20) for the baseband OFDM system. □

4.2. Link Budget

At $f_0 = 1.9$ GHz and for $d_0 = 100$ m, it is shown in [6] that $\kappa_{\rm dB} = 10\log_{10}(\kappa) \cong -78$ dB. From (1), we obtain $10\log_{10}\left(\frac{E}{\sigma_{\rm n}^2}\right) = \kappa_{\rm dB} + 10\gamma\log_{10}\left(\frac{d_0}{d}\right) + \mathrm{SNR}_{\rm dB}$, where $\mathrm{SNR}_{\rm dB}$ is the transmitted SNR in dB defined by $\mathrm{SNR}_{\rm dB} = 10\log_{10}\left(\frac{E_{\rm s}}{\sigma_{\rm n}^2}\right)$. In general, the term $10\log_{10}\left(\frac{E}{\sigma_{\rm n}^2}\right)$ can be considered as a received SNR. For a simple link budget, we assume $\gamma = 4.5425$, $d_0 = 100$ m and $d = 1{,}000$ m. If the receiver desires the received SNR, $10\log_{10}\left(\frac{E}{\sigma_{\rm n}^2}\right)$, of 0

[1] The result of (19) can also be verified in MATLAB® to yield (20).

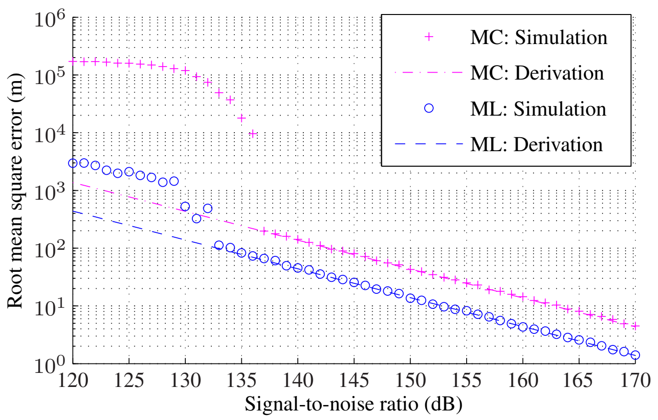


Fig. 1. RMSE of the position estimate as a function of the signal-to-noise ratio $\frac{E_s}{\sigma_n^2}$ (dB) for $\gamma = 4.5425$, $d = 1{,}000$ m, $T_s = 10^{-3}$ sec, $\bar{\beta} = 3.6517 \times 10^4$ Hz, sampling time $= 3.7037 \times 10^{-9}$ sec, and $N_R = 1{,}000$ independent runs.

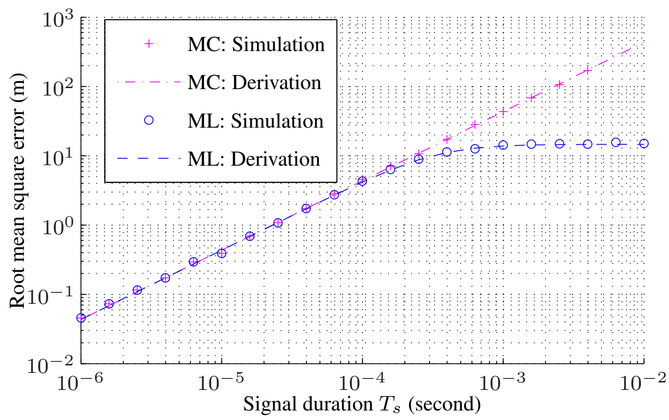


Fig. 2. RMSE of the position estimate as a function of the signal duration T_s (sec) for $\gamma = 4.5425$, $d = 1{,}000$ m, $10\log_{10}\left(\frac{E_s}{\sigma_n^2}\right) = 150$ dB, sampling time $= \frac{1}{2.5\times10^5}T$ sec, and $N_R = 1{,}000$ independent runs.

dB, the transmitter has to transmit the signal with $\text{SNR}_{\text{dB}} = 78.0168 - 45.425(2 - 3) = 123.4418$ dB. Considering the path attenuation, the transmitter should transmit a high SNR.

4.3. Maximum Correlation Implementation

To implement the cross-correlation in (10), we apply the Fouriér transform (see e.g. [15, p. 122]) such that

$$\begin{aligned}\rho(\tau) &= \Re\left(\int_{-\infty}^{\infty} r(t)s^*(-(\tau - t))\mathrm{d}t\right) \\ &= \Re\left(r(t) * s^*(-t)\right) \\ &= \Re\left(\int_{-\infty}^{\infty} R(f)S^*(f)\mathrm{e}^{j2\pi\tau f}\mathrm{d}f\right),\end{aligned} \tag{21}$$

where $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)\mathrm{d}\tau$ is the convolution between $f_1(t)$ and $f_2(t)$, and $R(f)$ is the Fouriér transform of $r(t)$. The cross-correlation can be efficiently computed by the fast Fouriér transform (FFT) of $r(t)$ and $s(t)$ and then the inverse FFT of their product.

4.4. Numerical Examples

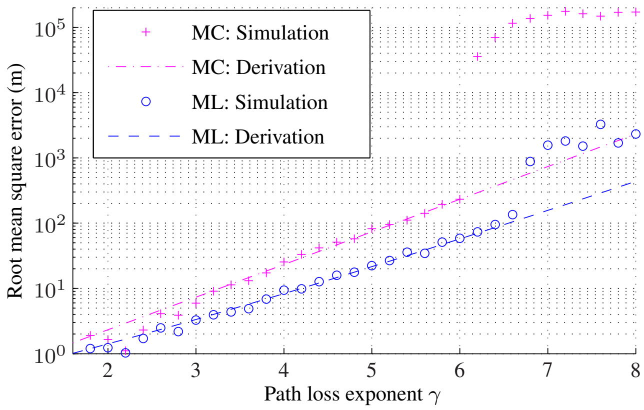
We employ the OFDM with $N = 2^5$ subbands. The quadrature phase shift keying (QPSK) is used as the signal constellation. Each component of the QPSK is independently drawn from $+1$ and -1 with equal probability. Theoretical root-mean-square error (RMSE) is computed from the square root of (12b) and (16b), which are multiplied by c. For the smallest computation, the observation period is chosen as $T = T_s + \tau_0$.

In Fig. 1, the RMSE of the estimate of the distance between the transmitter and the reciever is shown as a function of the transmitted SNR. For low SNR, both the MC and the ML provides meaningless distance estimates, i.e. the RMSE is larger than the actual distance. It can be seen that the SNR threshold, the SNR at which the ML and the MC fall into their asymptotic error, is approximately 137 dB for the MC and 133 dB for the ML. The use of the path loss can gain the accuracy of more than 600 m for a positioning system.

In Fig. 2, the RMSE is shown as a function of the signal duration T_s, which is inversely proportional to the effective bandwidth of the OFDM signal. It can be seen that when the signal duration is smaller, the RMSEs of both estimators approach the same value, i.e. $\frac{1}{8\pi^2\bar{\beta}^2\frac{E_s}{\sigma_n^2}a_0^2}$. For a larger signal duration, the ML provides a constant RMSE, which does not increase with the increase of the signal duration. The cause of this phenomenon is that the time delay in this regime is estimated mainly from the signal strength in the path gain. From (16b), the term $8\pi^2\bar{\beta}^2$ has less impact as the signal duration increases. The error saturation is then dominated by $\frac{1}{2\tau_0^2}\gamma^2$, which is independent of the effective bandwidth. The ML has a smaller RMSE than the MC. This is because the ML exploits the information of the path attenuation, while the MC has no information of the time delay in the path gain.

In Fig. 3, the RMSE is shown as a function of the path loss exponent γ. The RMSE increases with the larger value of γ. In (16b), the performance improvement of the path loss is gradually minor for a smaller γ. For a very severe situation, i.e. large γ, the received SNR drops such that both the MC and the ML cannot find the true ToA.

In Fig. 4, the RMSE is shown as a function of the distance d between the transmitter and the receiver. The large distance causes a larger estimation error. The performance improvement of the ML over the MC is more evident at a closer distance. For an extremely large distance, the received SNR drops again such that both the MC and the ML cannot



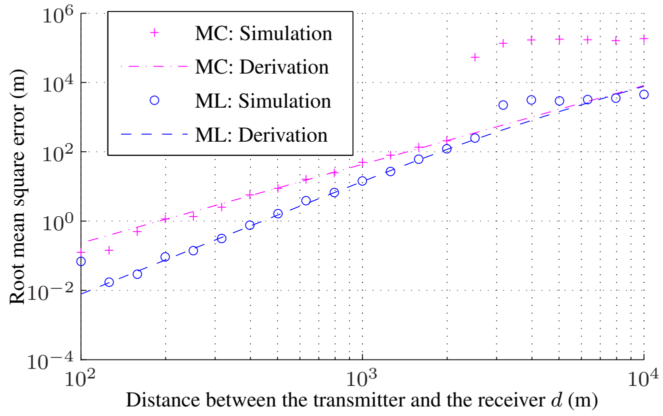


Fig. 3. RMSE of the position estimate as a function of the path loss exponent γ for $\bar{\beta} = 3.6517 \times 10^4$ Hz, $d = 1{,}000$ m and $10\log_{10}\left(\frac{E_s}{\sigma_n^2}\right) = 150$ dB, $T_s = 10^{-3}$ sec, sampling time $= \frac{1}{2\times10^6}T$ sec, and $N_R = 100$ independent runs.

Fig. 4. RMSE of the position estimate as a function of the distance d (m) for $\bar{\beta} = 3.6517 \times 10^4$ Hz, $\gamma = 4.5425$, $10\log_{10}\left(\frac{E_s}{\sigma_n^2}\right) = 150$ dB, $T_s = 10^{-3}$ sec, sampling time $= \frac{1}{2\times10^6}T$ sec, and $N_R = 100$ independent runs.

estimate the ToA.

5. CONCLUSION AND FUTURE DIRECTIONS

The exploration of the path loss increases the accuracy of the ToA estimation. The study of imperfect channel parameters, e.g. path loss exponent, is an upcoming work, while the effect of shadow fading was investigated in [16]. The path gain in (3) can be extended to include additional small-scale fading. Furthermore, multipath can also be taken in (2) into account.

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Time-of-Arrival Estimation in Path Attenuation

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Abstract

- We consider the time-of-arrival (ToA) estimation in the presence of path attenuation.
- Maximum correlation (MC) estimator is revisited and maximum likelihood (ML) estimator is newly derived to estimate the ToA.
- It reveals that for low effective bandwidth, short distance and large path loss exponent, the ML estimator has a smaller error variance than the MC estimator.
- Numerical examples illustrate that the ML estimator outperforms the MC estimator.

Introduction

- In the previous works, path gain is treated to be distance-independent and hence the ML solution yields an MC between the received signal and a delayed replica of the transmitted signal.
- Since the signal energy is attenuated during the propagation, the exploration of path loss in the path gain is a promising idea to improve the ToA estimation performance.
- The goal of this work is to investigate the benefit of deploying the attenuation of the path gain.

Received Energy

The received energy at the receiver can be expressed as (see, e.g., [Rappaport 2002, p. 38][a] and [Goldsmith 2005, p. 46][b])

$$E = \frac{d_0^{\gamma}}{d^{\gamma}} \kappa E_{\mathrm{s}}, \tag{1}$$

where d_0 is the close-in reference distance in the far field region, d is the distance between the receiver and the transmitter, γ is the path loss exponent, $E_{\mathrm{s}} = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ is the energy of transmitted signal $s(t)$, and κ is the unitless constant depending on antenna characteristics and average channel attenuation given by $\kappa = \frac{c^2}{16\pi^2 f_0^2 d_0^2}$, with the center frequency f_0 and the speed of light c.
Assuming $E = a^2 E_{\mathrm{s}}$, the path gain is given by

$$a_0 = \sqrt{\kappa} \left(\frac{d_0}{c\tau_0} \right)^{\frac{1}{2}\gamma}. \tag{2}$$

Transceiver Model

- The received baseband signal is

$$r(t) = a_0 s(t - \tau_0) + n(t), \tag{3}$$

where $s(t)$ is a known waveform, a_0 and τ_0 are the amplitude and the propagation time from the transmitter to the receiver, respectively, and $n(t)$ is an additive noise at the receiver and assumed to be a complex-valued zero-mean white Gaussian process with a variance of σ_{n}^2.

Maximum Correlation Estimator

The MC estimate of the ToA is given by (see, e.g., [Urkowitz 1983][c])

$$\hat{\tau}_{\mathrm{MC}} = \arg\max_{\tau} \int_0^T \Re\left(r^*(t) s(t-\tau) \right) \mathrm{d}t, \tag{4}$$

where T is the observation period, $\Re(\cdot)$ is the real part, and $(\cdot)^*$ is the complex conjugate. the bias and error variance of the MC estimate are written as

$$\mathrm{E}_{n(t)}\{\hat{\tau}_{\mathrm{MC}} - \tau_0\} = 0, \tag{5a}$$

$$\mathrm{E}_{n(t)}\{(\hat{\tau}_{\mathrm{MC}} - \tau_0)^2\} = \frac{1}{\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} 8\pi^2 \bar{\beta}^2 a_0^2}, \tag{5b}$$

where $\bar{\beta}$ is the effective (root-mean-square) bandwidth defined by

$$\bar{\beta} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 \mathrm{d}f}{\int_{-\infty}^{\infty} |S(f)|^2 \mathrm{d}f}}, \tag{6}$$

with $S(f)$ being the Fourier transform of $s(t)$.

[a][Rappaport 2002] T. S. Rappaport, *Wireless Communications: Principle and Practice*, 2 ed., Englewood Cliffs, NJ: Prentice Hall, 2002.
[b][Goldsmith 2005] A. Goldsmith, *Wireless Communications*, New York, NY: Cambridge University Press, 2005.
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Maximum Likelihood Estimator

The ML estimate of the ToA is given by

$$\hat{\tau}_{\mathrm{ML}} = \arg\min_{\tau} a^2(\tau) E_{\mathrm{s}} - 2a(\tau) \int_0^T \Re\left(r^*(t) s(t-\tau) \right) \mathrm{d}t. \tag{7}$$

The bias and error variance of the ML estimate are given by

$$\mathrm{E}_{n(t)}\{\hat{\tau}_{\mathrm{ML}} - \tau_0\} = 0, \tag{8a}$$

$$\mathrm{E}_{n(t)}\{(\hat{\tau}_{\mathrm{ML}} - \tau_0)^2\} = \frac{1}{\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} \left(8\pi^2 \bar{\beta}^2 + \frac{1}{2\tau_0^2} \gamma^2 \right) a_0^2}. \tag{8b}$$

Relative Performance

The ratio between the ML and the MC error variances is given by

$$\frac{\mathrm{E}_{n(t)}\{(\hat{\tau}_{\mathrm{ML}} - \tau_0)^2\}}{\mathrm{E}_{n(t)}\{(\hat{\tau}_{\mathrm{MC}} - \tau_0)^2\}} = \frac{1}{1 + \frac{1}{16\pi^2 \bar{\beta}^2 \tau_0^2} \gamma^2}. \tag{9}$$

Numerical Examples

The effective bandwidth of the baseband OFDM signal is given by

$$\bar{\beta} = \frac{1}{T_{\mathrm{s}}} \sqrt{\frac{1}{6}(2N^2 - 3N + 1)}, \tag{10}$$

where T_{s} is the signal duration, and N is the number of subbands.

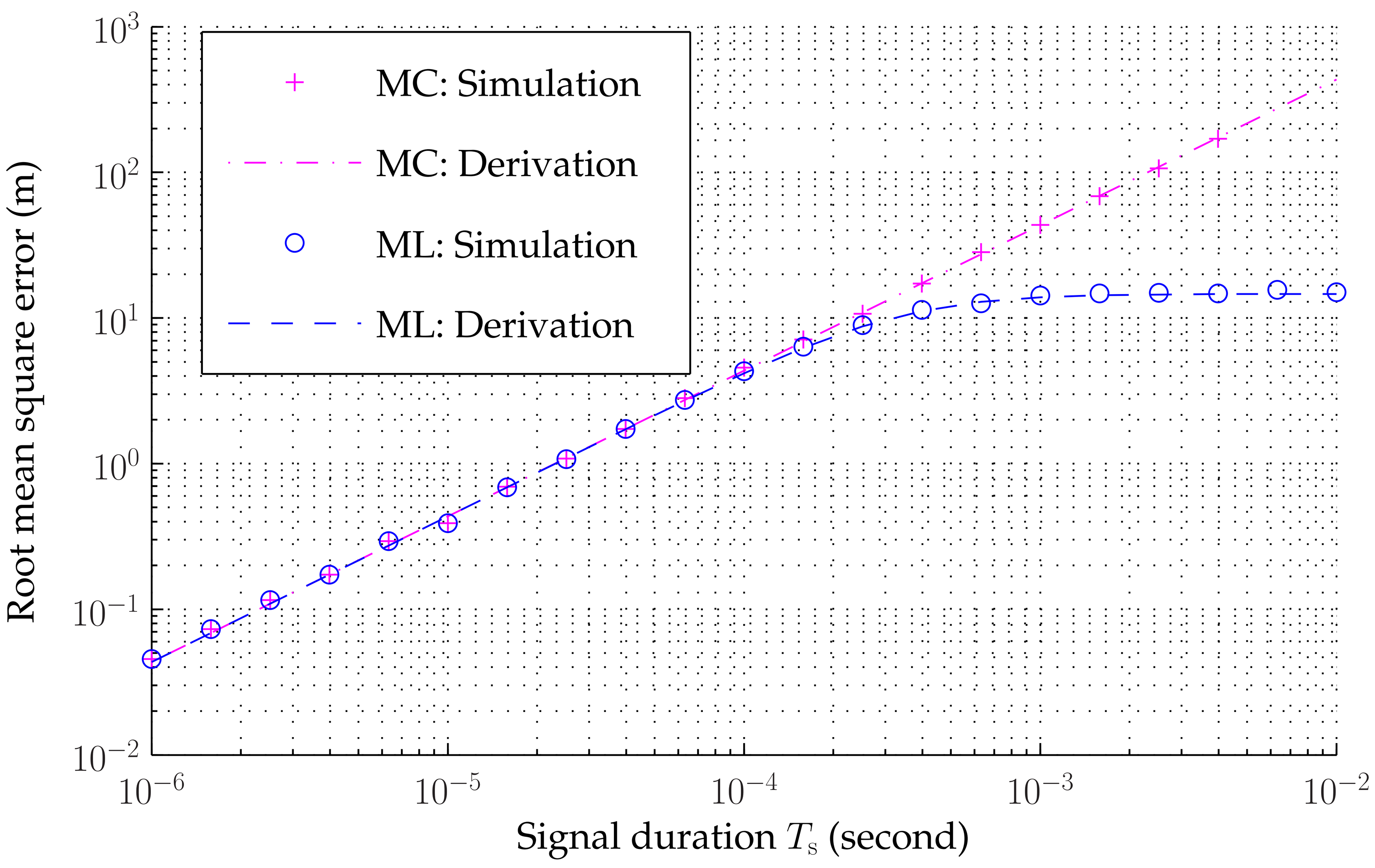

Fig. 1: RMSE of the position estimate as a function of the signal duration T_{s} (sec) for $\gamma = 4.5425$, $d = 1{,}000$ m, $10\log_{10}\left(\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\right) = 150$ dB, sampling time $= \frac{1}{2.5\times 10^5} T$ sec, and $N_{\mathrm{R}} = 1{,}000$ independent runs.

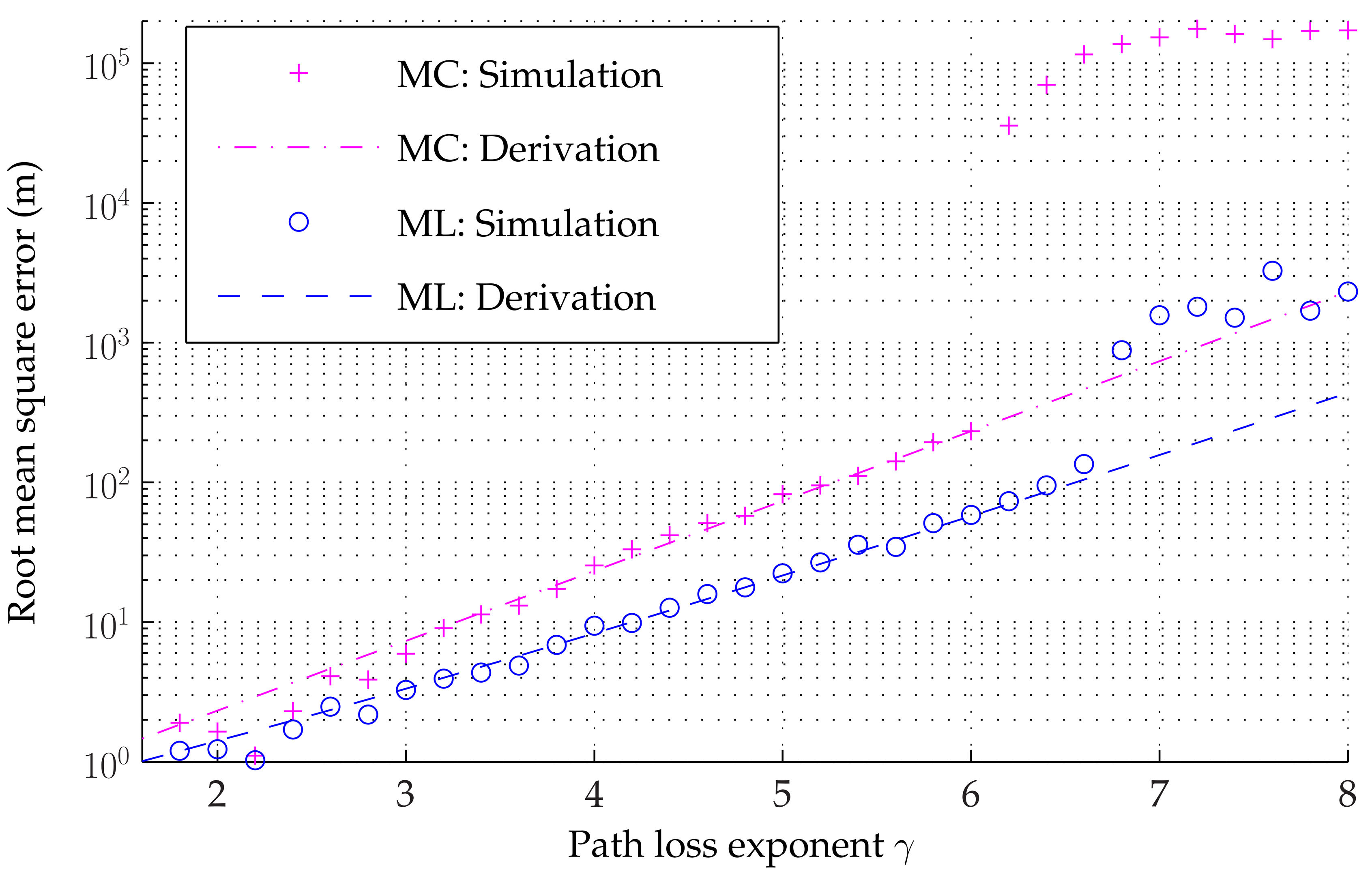

Fig. 2: RMSE of the position estimate as a function of the path loss exponent γ for $\bar{\beta} = 3.6517 \times 10^4$ Hz, $d = 1{,}000$ m and $10\log_{10}\left(\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\right) = 150$ dB, $T_{\mathrm{s}} = 10^{-3}$ sec, sampling time $= \frac{1}{2\times 10^6} T$ sec, and $N_{\mathrm{R}} = 100$ independent runs.