Intercarrier and Intersymbol Interference Analysis of OFDM Systems on Time-Varying channels

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Abstract— **In [2], intersymbol interference (ISI) and intercarrier interference (ICI) for OFDM systems with insufficient guard length was analyzed in the case of time-invariant radio channels. This was carried out by truncating the channel impulse response (CIR) in mathematical analysis into two parts, the so-called truncated channels. The same technique is used in this paper to compute the ISI and ICI components for the case of time-varying channels, where again the WSSUS assumption (wide sense stationary uncorrelated scattering) is applied. The exact expression of the ICI power is obtained in dependence of the time correlation function of the channel and the multipath channel profile. Moreover, this ICI power can be well approximated by the sum of the ICI power caused by the insufficient guard interval effect obtained in [2] and ICI power caused by the effect of time variations of the channel obtained in [4]. The ISI power is not dependent on the time variations of the channel.**

Keywords— **OFDM: ISI, ICI analysis.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is well known as the method to prevent the inter symbol interference (ISI) perfectly, when the length of the guard interval (GI) is longer than the maximal propagation delay of the channel. However, when the receiver moves from one transmission environment to another, e.g. indoor to outdoor like in HIPERLAN/2, the GI length condition may be no longer met.

The interference analysis for a time-invariant channel is completely introduced in [2]. This paper considers the more critical case that the GI length is insufficient and the channel is time-variant. In order to derive the mathematical description of interference and useful power in the case of insufficient guard length of an OFDM system, this paper uses the same approach as described in [2], that is the channel impulse response is truncated in mathematical analysis into two parts, the so-called truncated channels. The first one describes the behaviour inside the guard interval, the other outside. The linearity of the Fourier transform allows to analyze the effects of each truncated channel on the system separately. For simplification of theoretical analysis, the channel is assumed to be wide-sense stationary with uncorrelated scattering. We found that the intercarrier interference caused from other sub-carriers to the observed sub-carrier depends not only on the time correlation of the channel, but also on the multi-path channel profile. The ICI power can be well approximated by adding the ICI power caused by the insufficient GI effect on the time-invariant channel, and the ICI power caused by the time variations of the channel with sufficient GI length. This fact is observed by Steendam and Moeneclaey [5] in their computation results, and will be proved systematically in this paper.

In contrast to ICI power, the ISI power is independent of the time variations of the channel. That means the expression of ISI power for the case of time-varying channel is identical to that which was obtained for the time-invariant channel [2].

The paper is part II of [2] and organized as follows: The useful symbol and ICI contributions are studied in section II. Section III discusses the ISI contribution. Section IV presents numerical results, and section V concludes the paper.

II. Analysis of useful symbol, ICI contribution

Similar to [2], carrier and timing synchronization are assumed to be perfect and all analyses are considered in baseband. The expressions of ICI and ISI powers for the case of time-invariant channels are completely obtained in [2]. The demodulated symbol $\hat{d}_{l,i}$ on the l-th sub-carrier and the i-th OFDM symbol after taking the Fourier transform is given in Eq. (7) of [2] and is rewritten as follows for convenience:

$$\begin{aligned}\hat{d}_{l,i} &= \frac{1}{T_{\mathrm{S}}}\int\limits_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}}\Bigg\{\sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i}\int\limits_{\tau=0}^{\tau_{\max}} h(\tau,t)g(t-\tau-iT'_{\mathrm{S}})\\ &\quad\times e^{-j2\pi n f_{\mathrm{s}}\tau}d\tau\Bigg\}e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})}dt\\ &\quad+\frac{1}{T_{\mathrm{S}}}\sum_{i'=-\infty,i'\neq i}^{+\infty}\int\limits_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}}\Bigg\{\sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i'}\int\limits_{\tau=0}^{\tau_{\max}} h(\tau,t)\\ &\quad\times g(t-\tau-i'T'_{\mathrm{S}})e^{-j2\pi n f_{\mathrm{s}}\tau}d\tau\Bigg\}\\ &\quad\times e^{j2\pi f_{\mathrm{s}}[n(t-i'T'_{\mathrm{S}})-l(t-iT'_{\mathrm{S}})]}dt, \qquad (1)\end{aligned}$$

where T_{S}, T'_{S}, N_{C} are the OFDM symbol duration, the OFDM symbol duration plus GI, and the number of sub-carriers, respectively. The subscripts n, l denote the sub-carrier index, and i, i' represent the OFDM symbol index. $g(t)$ is the basic impulse of all sub-carriers defined in Eq. (2) of [2]. $f_{\mathrm{s}} = 1/T_{\mathrm{S}}$ is the sub-carrier spacing. The decomposion of the demodulated symbol in the general case is written as follows [2]:

$$\hat{d}_{l,i} = \hat{d}_{l,i}^{\mathrm{U}} + \hat{d}_{l,i}^{\mathrm{ICI-CIG}} + \hat{d}_{l,i}^{\mathrm{ICI-CTC}} + \hat{d}_{l,i}^{\mathrm{ISI}}, \quad (2)$$

where $\hat{d}_{l,i}^{\mathrm{U}}$, $\hat{d}_{l,i}^{\mathrm{ICI-CIG}}$, $\hat{d}_{l,i}^{\mathrm{ICI-CTC}}$ and $\hat{d}_{l,i}^{\mathrm{ISI}}$ are the useful symbol, the ICI contribution caused by the insufficient guard length, the ICI contribution caused by the time variations of the channel, and the ISI contribution, respectively. Similar to [2], the ISI analysis is separately studied in section III for simplification. The analysis results for the time-varying channel in case of sufficient guard length is shortly reviewed in section II-A. The new analytical results for the time-varying channel in case of insufficient guard length are derived in detail in section II-B.

A. Sufficient guard length

In this case, the ISI and the ICI-CIG contributions are not present. The expression of the ICI-CTC power is established by taking the autocorrelation of the ICI-CTC contribution. The final result of the calculation of the ICI-CTC power is obtained by Russel and Stüber [4]:

$$P_{\mathrm{ICI-CTC}} = \frac{E_{\mathrm{S}} \cdot E_{\mathrm{h}}}{T_{\mathrm{S}}^2} \sum_{n=0, n\neq l}^{N_{\mathrm{C}}-1} \int_{t=0}^{T_{\mathrm{S}}} \int_{t'=0}^{T_{\mathrm{S}}} R_t(t-t') \times e^{j2\pi(n-l)f_{\mathrm{s}}(t-t')} dt dt', \quad (3)$$

where $R_t(\Delta t), \Delta t = t - t'$, is the time-correlation function of the channel transfer function (CTF), and E_{h} is the channel energy which is normalized in [4]. The channel energy is calculated by $E_{\mathrm{h}} = \sum_{k=0}^{N_{\mathrm{P}}-1} \rho_k$, where ρ_k is the averaged energy of the tap k, and N_{P} is the number of taps of the channel. Now, the following section extends this result to the insufficient guard length case.

B. Insufficient guard length

In this case, all components in Eq. (2) must be taken into account. To analyze the effects of the part of the CIR within the guard interval and the part outside on the demodulated symbol, the CIR is truncated respectively into two parts. The first truncated channel $h_1(\tau, t)$ is the part within the guard interval of the CIR $h(\tau, t)$, and the second truncated channel $h_2(\tau, t)$ is the part outside[1]. The demodulated symbol expressed in Eq. (12) of [2] for time-invariant channels is now rewritten for time-varying channels as follows:

$$\begin{aligned}\hat{d}_{l,i} &= \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} H_1(nf_{\mathrm{s}}, t) e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})} dt \\ &+ \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \left\{ \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} \int_{\tau=T_{\mathrm{G}}}^{\tau_{\max}} h_2(\tau, t) \times g(t-\tau-iT'_{\mathrm{S}}) e^{\frac{-j2\pi n\tau}{T_{\mathrm{S}}}} d\tau \right\} e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})} dt \\ &+ \frac{1}{T_{\mathrm{S}}} \int_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} \sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i} H_2(nf_{\mathrm{s}}, t) \\ &\times e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})} dt + \hat{d}_{l,i}^{\mathrm{ISI}}, \end{aligned} \quad (4)$$

where $H_1(f, t)$ and $H_2(f, t)$ are the Fourier transforms of $h_1(\tau, t)$ and $h_2(\tau, t)$ with respect to τ, respectively. The useful symbol can be picked out from the first three terms of Eq. (4) by setting $n = l$. Then the integration bounds with respect to τ in the second term can be reduced since $g(t-\tau-iT'_{\mathrm{S}})$ is equal to zero in a certain interval and equal to one otherwise. Afterwards, the useful symbol is obtained as follows:

$$\begin{aligned}\hat{d}_{l,i}^{\mathrm{U}} &= \frac{d_{l,i}}{T_{\mathrm{S}}} \Bigg\{ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_1(lf_{\mathrm{s}}, t) dt \\ &+ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=T_{\mathrm{G}}}^{t-iT'_{\mathrm{S}}+T_{\mathrm{G}}} h_2(\tau, t) e^{-j2\pi l f_{\mathrm{s}}\tau} d\tau dt \\ &+ \int_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_2(lf_{\mathrm{s}}, t) dt \Bigg\}. \end{aligned} \quad (5)$$

The autocorrelation of $\hat{d}_{l,i}^{\mathrm{U}}$ is used to calculate the useful power P_{U}. While deriving its expression, the characteristics of the WSSUS channel are taken into account. It follows that the cross-correlation of the two truncated channels $H_1(f, t)$ and $H_2(f, t)$ vanishes. We obtain the final result as follows:

$$\begin{aligned}P_{\mathrm{U}} &= \frac{E_{\mathrm{S}}}{T_{\mathrm{S}}^2} \Bigg\{ E_{\mathrm{h}_1} \int_{t=0}^{T_{\mathrm{S}}} \int_{t'=0}^{T_{\mathrm{S}}} R_t(t-t') dt' dt \\ &+ E_{\mathrm{h}_2} \int_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} R_t(t-t') dt' dt \\ &+ 2 \int_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} \int_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=T_{\mathrm{G}}}^{t'+T_{\mathrm{G}}} \rho(\tau) R_t(t-t') d\tau dt' dt \\ &+ \int_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=T_{\mathrm{G}}}^{\min\{t+T_{\mathrm{G}}, t'+T_{\mathrm{G}}\}} \rho(\tau) \\ &\times R_t(t-t') d\tau dt' dt \Bigg\}, \end{aligned} \quad (6)$$

where $E_{\mathrm{h}_1} = \sum_{k=0}^{G-1} \rho_k$, and $E_{\mathrm{h}_2} = \sum_{k=G}^{N_{\mathrm{P}}-1} \rho_k$ are the energies of the impulse response of the first and the second truncated channel, respectively. G is the GI length in

[1]See Fig. 1 in [2].

samples. Equation (6) shows that *the average useful power in case of insufficient guard length depends not only on the time correlation function of the channel $R_t(\Delta t)$, but also the multi-path channel profile $\rho(\tau)$.*

The total ICI contribution is the sum of the ICI-CIG and ICI-CTC contributions, that is $\hat{d}_{l,i}^{\mathrm{ICI}} = \hat{d}_{l,i}^{\mathrm{ICI-CIG}} + \hat{d}_{l,i}^{\mathrm{ICI-CTC}}$. Bearing in mind Eq. (2), and comparing the expression of the useful symbol in (5) with the expression of the demodulated symbol in (4), it is readily seen that

$$\begin{aligned}\hat{d}_{l,i}^{\mathrm{ICI}} &= \frac{1}{T_{\mathrm{S}}} \sum_{n=0,n\neq l}^{N_{\mathrm{C}}-1} d_{n,i} \Bigg\{ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_1(nf_{\mathrm{s}},t) \\ &\times \; e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})}dt \\ &+ \int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}} \int_{\tau=T_{\mathrm{G}}}^{t-iT'_{\mathrm{S}}+\tau_{\max}} h_2(\tau,t)e^{-j2\pi nf_{\mathrm{s}}\tau}d\tau \\ &\times \; e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})}dt \\ &+ \int_{t=iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}^{iT'_{\mathrm{S}}+T_{\mathrm{S}}} H_2(nf_{\mathrm{s}},t)e^{j2\pi(n-l)f_{\mathrm{s}}(t-iT'_{\mathrm{S}})}dt \Bigg\},\end{aligned} \tag{7}$$

where, in the second term of (7), the integration bounds with respect to τ are changed to have $g(t-\tau-iT'_{\mathrm{S}})=1$.

The expression of the average total intercarrier interference power P_{ICI} is established by taking the autocorrelation of $\hat{d}_{l,i}^{\mathrm{ICI}}$. Similar to the way deriving the useful power, the cross-correlation functions of the two truncated channels $\mathrm{E}\{H_1^*(f,t)\cdot h_2(\tau,t)\}$ and $\mathrm{E}\{H_1^*(f,t)\cdot H_2(f,t)\}$ are also zero. This simplifies the expression of the average intercarrier interference power as follows

$$\begin{aligned}P_{\mathrm{ICI}} &= \frac{E_{\mathrm{S}}}{T_{\mathrm{S}}^2} \sum_{n=0,n\neq l}^{N_{\mathrm{C}}-1} \Bigg\{ E_{\mathrm{h_1}} \int_{t=0}^{T_{\mathrm{S}}}\int_{t'=0}^{T_{\mathrm{S}}} R_t(t-t') \\ &\times \; e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')}dtdt' \\ &+ \int_{t=0}^{\tau_{\max}-T_{\mathrm{G}}}\int_{t'=0}^{\tau_{\max}-T_{\mathrm{G}}}\int_{\tau=T_{\mathrm{G}}}^{\min\{\mathrm{t+T_G,t'+T_G}\}} \rho(\tau)R_t(t-t')d\tau \\ &\times \; e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')}dtdt' \\ &+ \; 2\int_{t=0}^{\tau_{\max}-T_{\mathrm{G}}}\int_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}}\Bigg[\int_{\tau=T_{\mathrm{G}}}^{t+T_{\mathrm{G}}} \rho(\tau)R_t(t-t')d\tau \\ &\times \; e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')}dt'dt \\ &+ \; E_{\mathrm{h_2}}\int_{t=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}}\int_{t'=\tau_{\max}-T_{\mathrm{G}}}^{T_{\mathrm{S}}} R_t(t-t') \\ &\times \; e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')}dtdt'\Bigg\}.\end{aligned} \tag{8}$$

The first term of (8) is completely the interference power caused by the time variations of the first truncated channel, whereas the last three terms describes the interference power caused by the second truncated channel including the effect of insufficient guard interval length and the time variations of the channel. Examination of (8) reveals that *the average ICI power depends on the time-correlation function of the channel transfer function and the multi-path channel profile.*

The expression of average ICI power in (8) looks somehow cumbersome, but it is well approximated by adding the average ICI-CTC power in Eq. (3) given by Russel and Stüber [4] and the average ICI-CIG power given in Eq. (24) [2]:

$$P_{\mathrm{ICI}} \approx P_{\mathrm{ICI-CTC}} + P_{\mathrm{ICI-CIG}}. \tag{9}$$

Then the approximated average ICI power is larger than the exact value, but the approximation error can be neglected if $T_{\mathrm{S}} \ll (\Delta t)_c$ or $T_{\mathrm{S}} \gg \tau_{\max} - T_{\mathrm{G}}$, where $(\Delta t)_c$ is the coherence time of the channel. Usually, both conditions are fulfilled in a well-designed OFDM system. Then the average ICI-CIG power is further approximated in Eq. (25) of [2]. It results in the expression of the approximated ICI power as follows:

$$\begin{aligned}P_{\mathrm{ICI}} &\approx \frac{E_{\mathrm{S}}\cdot E_{\mathrm{h}}}{T_{\mathrm{S}}^2}\sum_{\substack{n=0\\n\neq l}}^{N_{\mathrm{C}}-1}\int_{t=0}^{T_{\mathrm{S}}}\int_{t'=0}^{T_{\mathrm{S}}} R(t-t')e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')} \\ & dtdt' \\ &+ \; \frac{E_{\mathrm{S}}}{T_{\mathrm{S}}}\int_{t=0}^{\tau_{\max}-T_{\mathrm{G}}}\int_{\tau=t+T_{\mathrm{G}}}^{\tau_{\max}} \rho(\tau)d\tau dt.\end{aligned} \tag{10}$$

The first term of Eq. (10) is the ICI power caused by the effect of the time variations of the channel, and the second term is the ICI power caused by the effect of insufficient guard interval.

III. Analysis of ISI

The description of $\hat{d}_{l,i}^{\mathrm{ISI}}$ is identical to that which was derived in [2], however the channel is now time-varying. Therefore Eq. (27) in [2] is rewritten as follows:

$$\begin{aligned}\hat{d}_{l,i}^{\mathrm{ISI}} &= \frac{1}{T_{\mathrm{S}}}\int_{t=iT'_{\mathrm{S}}}^{iT'_{\mathrm{S}}+\tau_{\max}-T_{\mathrm{G}}}\Bigg\{\sum_{n=0}^{N_{\mathrm{C}}-1} d_{n,i-1}\int_{\tau=T_{\mathrm{G}}+t-iT'_{\mathrm{S}}}^{\tau_{\max}} \\ & h_2(\tau,t)e^{-j2\pi nf_{\mathrm{s}}\tau}d\tau\Bigg\}e^{j2\pi f_{\mathrm{s}}[(n-l)(t-iT'_{\mathrm{S}})+nT'_{\mathrm{S}}]}dt.\end{aligned} \tag{11}$$

Assuming that the number of sub-carriers is equal to the FFT length, i.e. $N_{\mathrm{C}} = N_{\mathrm{FFT}}$, then proceeding similar as in [2], we obtain the final expression of the average ISI power as follows:

$$P_{\mathrm{ISI}} = \frac{E_{\mathrm{S}}}{T_{\mathrm{S}}} \int\limits_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=T_{\mathrm{G}}+t}^{\tau_{\max}} \rho(\tau,0) d\tau dt, \qquad (12)$$

where $\rho(\tau, \Delta t)$ is the autocorrelation function of the CIR $h(\tau, t)$ [3]. Letting $\Delta t = 0$, the autocorrelation function of the CIR becomes the multi-path channel profile: $\rho(\tau) = \rho(\tau, 0)$. Comparing with Eq. (29) in [2], we see that *the average ISI power is not dependent on the time variations of the channel.* With the approximation of the ICI power in Eq. (10), and the exact calculation of ISI power in Eq. (12), the total interference power can be approximated as follows:

$$\begin{aligned} P_{\mathrm{I}} \approx\ & \frac{E_{\mathrm{S}} \cdot E_{\mathrm{h}}}{T_{\mathrm{S}}^2} \sum_{\substack{n=0 \\ n \neq l}}^{N_{\mathrm{C}}-1} \int\limits_{t=0}^{T_{\mathrm{S}}} \int\limits_{t'=0}^{T_{\mathrm{S}}} R(t-t') e^{-j2\pi(n-l)f_{\mathrm{s}}(t-t')} \\ & dt dt' \\ + & \frac{2E_{\mathrm{S}}}{T_{\mathrm{S}}} \int\limits_{t=0}^{\tau_{\max}-T_{\mathrm{G}}} \int\limits_{\tau=t+T_{\mathrm{G}}}^{\tau_{\max}} \rho(\tau) d\tau dt. \end{aligned} \qquad (13)$$

It shows that the range of $-T_{\mathrm{S}} \leq \Delta t = t - t' \leq T_{\mathrm{S}}$ of the time-correlation function of the CTF, $R(\Delta t)$, determines the interference power caused by the effect of the time variations of the channel, and the range of $T_{\mathrm{G}} \leq \tau \leq \tau_{\max}$ of the multi-path channel profile determines the interference power caused by the effect of insufficient guard length.

IV. Numerical results

The system parameters and the channel model described in [2] are used for computations. The system parameters are basically taken from HIPERLAN/2 specified in [1], however the number of sub-carriers is assigned to be the FFT length ($N_{\mathrm{C}} = N_{\mathrm{FFT}} = 64$) according to the theoretical analysis assumption. The maximal Doppler frequency is selected to be 1000 Hz with the purpose that the the FFT length does not require to be chosen too large to see the effect of the time variations of the channel. Otherwise the computation time would be rather long.

Due to the loss of orthogonality caused by the fading channel and the insufficient guard length, the average power P_{T} at the output of the FFT consists of the average useful power and the average interference power and is decomposed as follows:

$$P_{\mathrm{T}} = P_{\mathrm{U}} + P_{\mathrm{ICI}} + P_{\mathrm{ISI}}, \qquad (14)$$

where P_{U}, P_{ICI} and P_{ISI} are computed from Eqs. (6), (8) and (12), respectively. It can be proved that P_{T} is always equal to the product of the symbol energy E_{S} and the energy of the channel impulse response E_{h}, i.e. $P_{\mathrm{T}} = E_{\mathrm{S}} \cdot E_{\mathrm{h}}$. Thus, the average interference power P_{I} is calculated by

$$P_{\mathrm{I}} = P_{\mathrm{ICI}} + P_{\mathrm{ISI}} = P_{\mathrm{T}} - P_{\mathrm{U}}. \qquad (15)$$

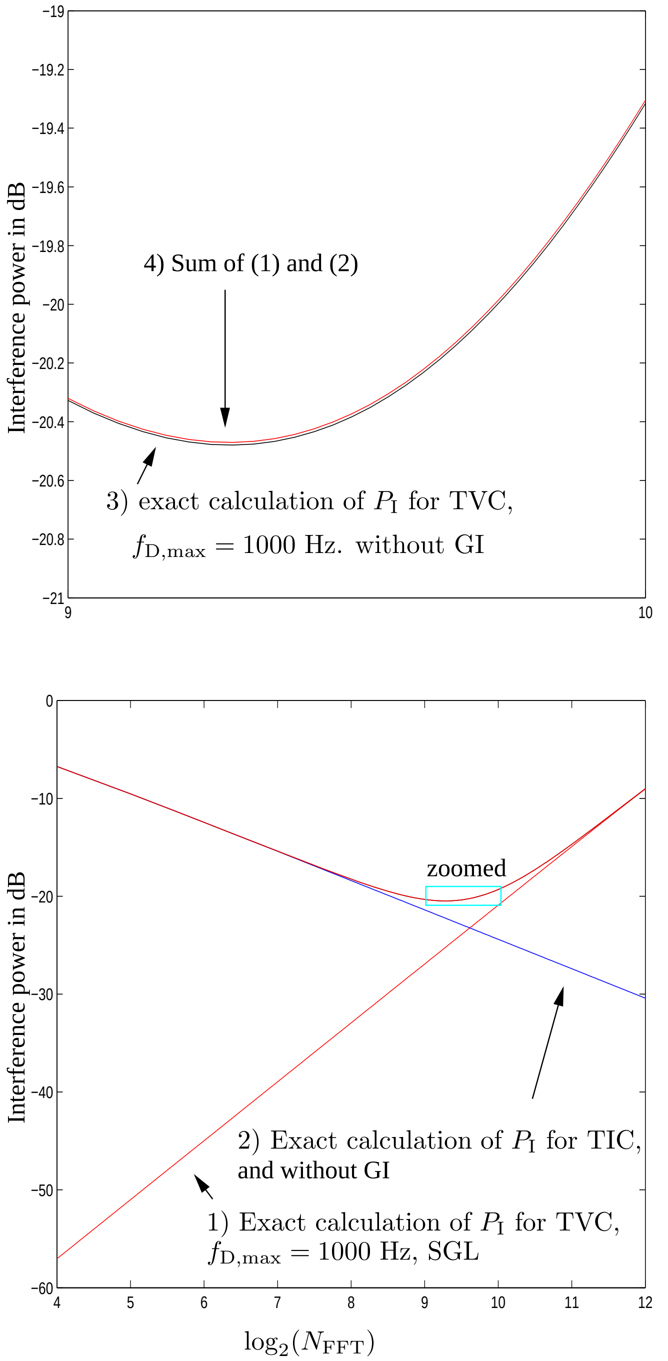

Fig. 1. Interference power introduced by the effects of time variations of the channel and insufficient guard length is well approximated by the sum of interference power introduced merely by the effect of time variations of the channel with sufficient GI length, and interference power introduced merely by the insufficient GI length effect on the time-invariant channel. Upper diagram: Zoomed part as marked in the lower diagram.

In order to obtain an exact calculation of P_{I}, only the calculation of P_{U} is required instead of calculations of the formulas (8) and (12). The results of interference computation as a function of the FFT length are plotted in Fig. 1 for four different cases as follows:

- Case 1: Exact calculation of interference power for the time-varying channel (TVC) and sufficient guard length (SGL) case. The interference power in this case is $P_{\mathrm{ICI-CTC}}$ given in Eq. (3).
- Case 2: Exact calculation of interference power for the time-invariant channel (TIC) and without guard interval. It is the sum of $P_{\mathrm{ICI-CIG}}$ in Eq. (24) and P_{ISI} in Eq. (29) of [2].
- Case 3: Exact calculation of interference power for the time-varying channel and without guard interval. It can be calculated by the sum of P_{ICI} in Eq. (8) and P_{ISI} in Eq. (12), or it can be more effectively calculated according to its expression in Eq. (15).
- Case 4: Sum of case (1) and (2).

Case 1 is analyzed by Russel and Stüber [4], case 2 can be seen from [2]. Both of them are given for reference. From case 3, we see that increasing of FFT length, as long as the time variations of the channel are still insignificant, leads to decreasing of interference power. This is because the ISI and ICI-CIG powers are inversely proportional to the FFT length. However, with increasing of FFT length, the effect of the time variations of the channel increases proportionally. If the ICI-CTC power becomes noticeable, that is the OFDM symbol duration becomes considerable compared to the coherence time of the channel, the total interference power starts to increase. This is because the dominant interference contribution has changed to ICI-CTC power. If we add the interference power of case (1) to case (2), it leads to the good approximation of the interference power resulting from effects of time variations of the channel and insufficient guard interval length. This can be seen by comparing the result of case (3) with case (4) in Fig. 1. As mentioned in section III, the intersymbol interference powers in case (2) and case (3) are equal. Thus the approximation of the intercarrier interference power in Eq. (9) is verified.

Since the ICI-CIG power $P_{\mathrm{ICI-CIG}}$ in Eq. (24) of [2] can be approximated in Eq. (25) of [2], the total interference power for an OFDM system on a time-varying channel and in the case of insufficient GI length is further approximated in Eq. (13). This approximation is also verified by computations as follows: We consider the interference powers of four different cases as described as above. But in case (2), the approximated expression of $P_{\mathrm{ICI-CIG}}$ in Eq. (25) of [2] is used for computation. The computation results shown in Fig. 2 demonstrate the validity of this approximation. However, in the range where the FFT length is small ($2^4 \rightarrow 2^5$), the condition $T_{\mathrm{S}} \gg \tau_{\max} - T_{\mathrm{G}}$ for the approximation of $P_{\mathrm{ICI-CIG}}$ is not fulfilled. Thus, the approximation error can be seen in this range.

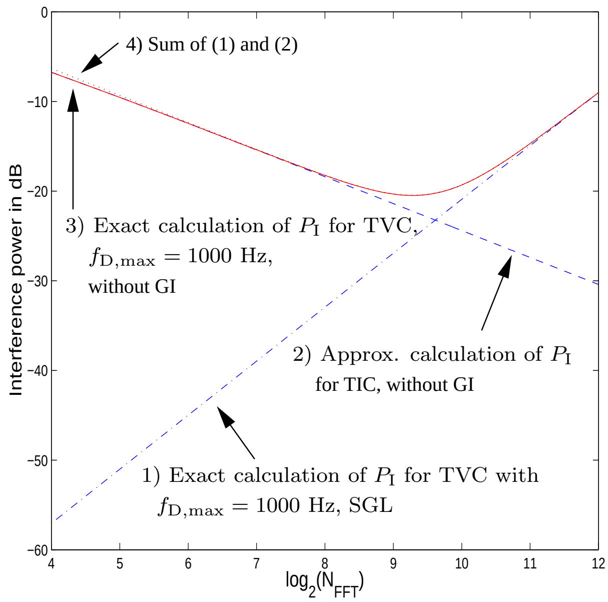

Fig. 2. Verification of the approximation of the total interference power in Eq.(13).

V. Conclusions

In case of a time-varying channel and insufficient guard length, ICI consists of two contributions, where one is caused by the time variations of the channel (ICI-CTC), and the other is caused by the insufficient guard length (ICI-CIG). By truncating the channel impulse response into two parts as introduced in [2], we have obtained exactly the mathematical expressions of the ISI and ICI powers for the time-varying channel if the guard interval length is insufficient. Interestingly, the interference power in this case is well approximated by adding the interference power caused merely by the effect of the time variations of the channel and the interference power caused merely by the effect of insufficient guard interval length.

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