INTERCARRIER AND INTERSYMBOL INTERFERENCE ANALYSIS OF OFDM SYSTEMS ON TIME-INVARIANT CHANNELS

Van Duc Nguyen, Hans-Peter Kuchenbecker

University of Hannover, Institut für Allgemeine Nachrichtentechnik
Appelstr. 9A, D-30167 Hannover, Germany
Phone: +49-511-762-2842, E-mail: nguyen@ant.uni-hannover.de

Abstract— **In order to derive the mathematical description of interference and useful power in the case of insufficient guard length of an OFDM system, the channel impulse response is truncated in mathematical analysis into two parts, the so-called two truncated channels. The first one describes the behaviour inside the guard interval, the other outside. Assuming the channel to be wide-sense stationary with uncorrelated scattering (WSSUS), the linear characteristic of Fourier transformation allows to obtain the mathematical expressions of intercarrier interference (ICI) power P_{ICI}, intersymbol interference (ISI) power P_{ISI} and useful power P_{U} in dependence of the correlation functions and the multi-path channel profiles of two truncated channels exactly. In this paper, we consider only the case of the time-invariant channel. The simulation results show a good agreement with the theoretical results.**

Keywords— **OFDM: ISI, ICI analysis.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multi carrier modulation technique, which is well known as the method to prevent the Inter Symbol Interference (ISI) perfectly, when the guard interval is longer than the maximal propagation delay. However, when the receiver moves from one transmission environment to another , e.g. indoor to outdoor like in HIPERLAN/2, the guard length condition may be no longer met.

In the mobile transmission environment, the Channel Transfer Function (CTF) of the mobile channel is generally time- and frequency-selective. The time-selectivity caused by the movement of the receiver affects the orthogonality of the carriers and therefore introduces ICI at the receiver. The ICI caused by the time-varying channel (ICI-CTC) decreases the performance of the system, and increases proportionally to the product of the Doppler spread and the OFDM symbol duration ($f_d T_S$).

In literature, Russel and Stüber [3] obtained exact expressions of ICI-CTC power resulting from Doppler spread for an OFDM system in the case of sufficient guard length. However, in their study, they did not analyse the case of insufficient guard length.

The ICI is also introduced even in the case of time-invariant channel if the maximum delay spread of the channel exceeds the guard interval length. We named this ICI term ICI-CIG (ICI caused by insufficient guard length). In order to derive the mathematical description of different interference contributions as well as useful contribution, a novel method for the analysis of interference contributions is proposed. This method truncates the channel impulse response $h(\tau, t)$ in mathematical analysis into two parts, the so-called two truncated channels: $h_1(\tau, t)$ and $h_2(\tau, t)$. In our study, we derived that $h_1(\tau, t)$ causes no ISI but a part of the total ICI-CTC and $h_2(\tau, t)$ causes the ISI, a part of the total ICI-CTC and also the ICI-CIG. Throughout this paper, when the ICI is mentioned, this means both the ICI-CTC and the ICI-CIG.

P_{ISI} contributed by $h_2(\tau, t)$ depends mainly on the tail outside the guard interval of the multi path channel profile $\rho(\tau, \Delta t)$. P_{ICI} consists of two contributions: $P_{\mathrm{ICI}} = P_{\mathrm{ICI-CTC}} + P_{\mathrm{ICI-CIG}}$, where $P_{\mathrm{ICI-CTC}}$ caused by the time variation of the channel depends on the time correlation function of the channel, whereas $P_{\mathrm{ICI-CIG}}$, similar to P_{ISI}, is dependent on the tail outside the guard interval of the multi-path channel profile.

For brevity, our research is divided in two parts. Part I describes the analysis results for the time-invariant channel and is published in this paper. Part II describes the analysis results for the time-varying channel and is in preparation. The organisation of this paper is as follows: Section II gives the system description, section III focuses on the analysis of ICI and useful signal. The analysis of ISI in case of time-invariant channel is studied in section IV. Section V presents numerical results. Finally, section VI presents some concluding remarks.

II. System description

Throughout this paper, carrier and timing synchronization are assumed to be perfect, all analyses are considered in baseband. Although the realisation of OFDM systems is based on discrete samples, the analysis is here in continuous form. We consider an input data symbol sequence $(d_{0,i}, d_{1,i}, ..., d_{n,i}, ...d_{N_C-1,i})$ with the OFDM symbol index i and the sub-carrier index n. N_C is the number of sub-carriers. The transmitted OFDM symbol can be generated by using a N_{FFT}-point IFFT. The guard interval is a cyclic extension of the IFFT output, so that the i-th transmitted OFDM symbol with guard interval is

$$x_i(t) = \frac{1}{T_S} \sum_{n=0}^{N_C-1} d_{n,i} g(t - iT'_S) e^{\frac{j2\pi n(t-iT'_S)}{T_S}}$$

$$-T_G + iT'_S \leq t < iT'_S + T_S, \quad -\infty \leq i \leq \infty \qquad (1)$$

where T_G, T_S and $T'_S = T_G + T_S$ are the guard interval duration, the OFDM symbol duration without and with guard interval, respectively. $g(t)$ is the basic impulse of all sub-carriers, given as follows:

$$g(t) = \begin{cases} k & : \text{ for } \quad -\mathrm{T_G} \leq \mathrm{t} \leq \mathrm{T_S} \\ 0 & : \text{ otherwise} \end{cases} \qquad (2)$$

where k is simply a constant factor, and throughout this paper it is assumed to be 1 without loosing generality. If there is no additive noise during the transmission, the received signal after traveling through the multi-path fading channel is given by

$$y(t) = x(t) \star h(\tau, t) \qquad (3)$$

where the symbol $'\star'$ represents the convolution of the transmitted signal $x(t) = \sum_{i=-\infty}^{+\infty} x_i(t)$ with the time-varying channel impulse response $h(\tau, t)$ with respect to τ. After a further analysis of the received signal in Eq. (3), taking into account the sole transmitted signal of the i-th OFDM symbol, we obtain the received signal as follows:

$$y_i(t) = \frac{1}{T_S} \sum_{n=0}^{N_C-1} d_{n,i} \int_0^{\tau_{max}} h(\tau, t) g(t - \tau - iT'_S) \times e^{\frac{j2\pi n(t-\tau-iT'_S)}{T_S}} d\tau \qquad (4)$$

Now if we truncate $h(\tau, t)$ in mathematical analysis as shown in Fig. 1 into two channels $h_1(\tau, t)$ and $h_2(\tau, t)$, the expression of $y_i(t)$ in Eq. (4) yields

$$y_i(t) = \underbrace{x_i(t) \star h_1(\tau, t)}_{y_{i_1}(t)} + \underbrace{x_i(t) \star h_2(\tau, t)}_{y_{i_2}(t)}, \qquad (5)$$

where $h_1(\tau, t)$ and $h_2(\tau, t)$ are depicted in Fig.1.b and Fig.1.c, respectively. Figure 1.d, 1.e and 1.f illustrate the transmitted signal, the received signal caused by the first truncated channel and the second truncated channel, respectively. In Eq. (5), the contribution of the received signal $y_{i_1}(t)$ causes no ISI because the duration of $h_1(\tau, t)$ is not longer than the guard interval. On the contrary, the energy of $h_2(\tau, t)$ situated outside guard interval, influences the following symbol, thus $y_{i_2}(t)$ contributes to the intersymbol interference in the following symbol corresponding to area IV in Fig. 1.f. Moreover , the FFT of $y_{i_2}(t)$ belonging to area I and II yields two different terms: The first term is the wanted symbol distorted by the multiplicative distortion derived from $H_2(f, t)$ and $h_2(\tau, t)$ (see later in subsection III-B), second term is distortion and corrupts the wanted data symbol on sub-carriers in the current OFDM symbol. Therefore, the second term is regarded as the ICI-CIG, which is understood as the distortion caused by the adjacent sub-carriers to the observed sub-carrier. In the next section we concentrate on the analysis of ISI and ICI as well as on the useful signal contributions at the output of the OFDM demodulator for different cases of the guard length condition.

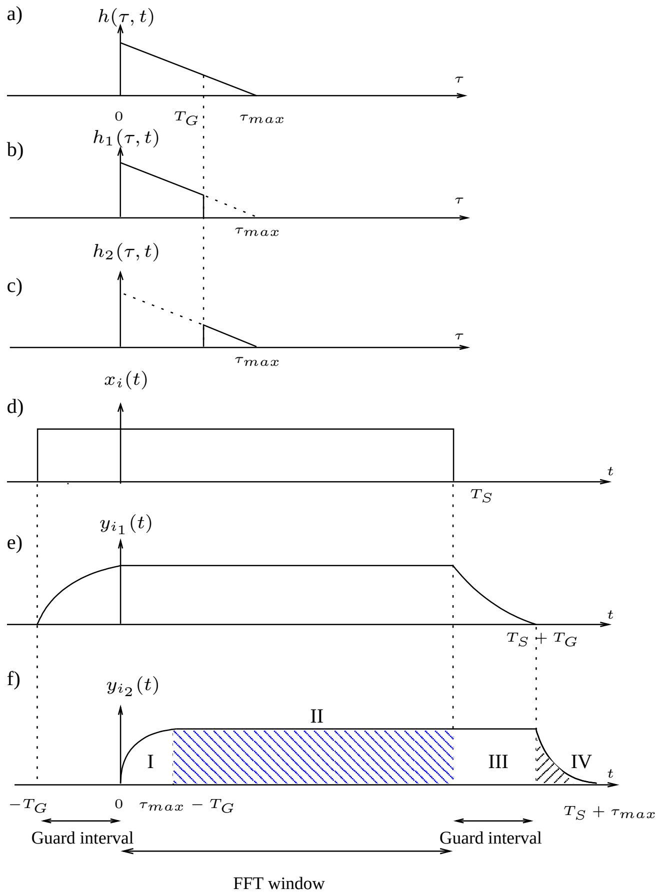

Fig. 1. The received OFDM signal disturbed by insufficient guard interval (illustrated for i=0)

III. Analysis of useful symbol and ICI contributions

Let us now study the demodulated symbol $\hat{d}_{l,i}$ at the sub-carrier frequency lf_s during the i-th OFDM symbol period, where $f_s = \frac{1}{T_S}$ is the sub-carrier spacing. After removing the guard interval, the l-th output of the OFDM demodulator in analog form is the Fourier transformation of the received signal $y(t) = \sum_{i=-\infty}^{+\infty} y_i(t)$ applied in the integration interval $t \in [iT'_S, iT'_S + T_S]$.

$$\hat{d}_{l,i} = \int_{t=iT'_S}^{iT'_S+T_S} y_i(t) e^{\frac{-j2\pi l(t-iT'_S)}{T_S}} dt$$

$$+\sum_{i'=-\infty,i'\neq i}^{+\infty}\int_{t=iT'_S}^{iT'_S+T_S} y_{i'}(t)e^{\frac{-j2\pi l(t-iT'_S)}{T_S}}dt \quad (6)$$

Equation (6) describes the different kinds of contributions to the demodulated symbol $\hat{d}_{l,i}$, where the first term consists of the useful symbol including ICI, the second term is obviously the ISI. After substituting $y_i(t)$ and $y_{i'}(t)$ from Eq. (4) in (6), the demodulated symbol becomes

$$\hat{d}_{l,i}=\frac{1}{T_S}\int_{t=iT'_S}^{iT'_S+T_S}\left\{\sum_{n=0}^{N_C-1}d_{n,i}\int_{\tau=0}^{\tau_{max}}h(\tau,t)g(t-\tau-iT'_S)\right.$$
$$\left.\times e^{\frac{-j2\pi n\tau}{T_S}}d\tau\right\}e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt$$
$$+\frac{1}{T_S}\sum_{i'=-\infty,i'\neq i}^{+\infty}\int_{t=iT'_S}^{iT'_S+T_S}\left\{\sum_{n=0}^{N_C-1}d_{n,i'}\int_{\tau=0}^{\tau_{max}}h(\tau,t)\right.$$
$$\left.\times g(t-\tau-i'T'_S)e^{\frac{-j2\pi n\tau}{T_S}}d\tau\right\}$$
$$\times e^{\frac{j2\pi[n(t-i'T'_S)-l(t-iT'_S)]}{T_S}}dt \quad (7)$$

The contributions in $\hat{d}_{l,i}$ depend on different cases of guard length condition (sufficient length or insufficient length) and channel model (time-invariant or time-variant). In general case, $\hat{d}_{l,i}$ can be decomposed as

$$\hat{d}_{l,i}=\hat{d}^{\mathrm{U}}_{l,i}+\hat{d}^{\mathrm{ICI-CIG}}_{l,i}+\hat{d}^{\mathrm{ICI-CTC}}_{l,i}+\hat{d}^{\mathrm{ISI}}_{l,i} \quad (8)$$

where $\hat{d}^{\mathrm{U}}_{l,i}$, $\hat{d}^{\mathrm{ICI-CIG}}_{l,i}$, $\hat{d}^{\mathrm{ICI-CTC}}_{l,i}$ and $\hat{d}^{\mathrm{ISI}}_{l,i}$ are the useful symbol, the ICI contribution caused by the insufficient guard length, the ICI contribution caused by the time variations of the channel, and the ISI contribution, respectively. The ISI analysis is separately studied in section IV. In order to derive the mathematical descriptions of the other contributions, the sufficient guard length and insufficient guard length cases are considered. In this paper, only the time-invariant channel (i.e. $h(\tau,t)=h(\tau)$) is considered. In the following, the theoretical analysis is investigated in detail.

A. Sufficient guard length: $T_G \geq \tau_{max}$

In the sufficient guard length case, it is easy to see that $h_2(\tau)$ is zero in Fig. 1, therefore there is no ICI-CIG and ISI. Thus, equation (8) is shortly rewritten as:

$$\hat{d}_{l,i}\Big|_{T_G\geq\tau_{max}}=\hat{d}^{\mathrm{U}}_{l,i}+\hat{d}^{\mathrm{ICI-CTC}}_{l,i} \quad (9)$$

The first term of Eq. (9) denotes the useful symbol and can be straightforwardly written as

$$\hat{d}^{\mathrm{U}}_{l,i}\Big|_{T_G\geq\tau_{max}}=d_{l,i}H(lf_s) \quad (10)$$

where $H(lf_s)$ is the sample taken at the l-th sub-carrier frequency of the channel transfer function $H(f)$. Since the orthogonality between sub-carriers is fulfilled, the second term of Eq. (9) is zero: $\hat{d}^{\mathrm{ICI-CTC}}_{l,i}=0$. Therefore, the demodulated symbol is also the useful symbol. In this case, the transmitted symbol is completely recovered by multiplying the demodulated symbol by the channel coefficient $H^{-1}(lf_s)$, if the channel is perfectly known at the receiver.

B. Insufficient guard length: $T_G < \tau_{max}$

In this case, while calculating the demodulated symbol $\hat{d}_{l,i}$, ISI and ICI-CIG according to areas IV, I and II in Fig. 1.f must be taken into account. According to Fig. 1.b and 1.c, the integration with respect to the variable τ in Eq. (7) is divided in two periods. The first period is within $0\leq\tau\leq T_G$ and the second period is within $T_G<\tau\leq\tau_{max}$, where the first truncated channel $h_1(\tau)$ and the second truncated channel $h_2(\tau)$ are located, respectively. Then, Eq.(7) is rewritten as

$$\hat{d}_{l,i}=\frac{1}{T_S}\int_{t=iT'_S}^{iT'_S+T_S}\left\{\sum_{n=0}^{N_C-1}d_{n,i}\int_{\tau=0}^{T_G}h_1(\tau)g(t-\tau-iT'_S)\right.$$
$$\left.\times e^{\frac{-j2\pi n\tau}{T_S}}d\tau\right\}e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt+\frac{1}{T_S}\int_{t=iT'_S}^{iT'_S+T_S}$$
$$\left\{\sum_{n=0}^{N_C-1}d_{n,i}\int_{\tau=T_G}^{\tau_{max}}h_2(\tau)g(t-\tau-iT'_S)e^{\frac{-j2\pi n\tau}{T_S}}d\tau\right\}$$
$$\times e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt+\hat{d}^{\mathrm{ISI}}_{l,i} \quad (11)$$

In the first term of Eq. (11), it can be verified that $g(t-\tau-iT'_S)=1$ for all t and τ in the integration bounds. Thus, the integration result with respect to τ is obviously the CTF of the first truncated channel $H_1(nf_s)$ on n-th sub-carrier. To analyse the second term of Eq. (11), we separate the integration with respect to t into two intervals. The first integration interval is $iT'_S\leq t< iT'_S+\tau_{max}-T_G$, which corresponds to area I in Fig. 1.f. The second integration interval is $iT'_S+\tau_{max}-T_G\leq t\leq iT'_S+T_S$, which corresponds to the area II in Fig. 1.f. In the second integration interval, it is straightforward to confirm, that $g(t-\tau-iT'_S)=1$ for $\forall\tau\in(T_G,\tau_{max})$. Therefore, equation (11) can be represented as

$$\hat{d}_{l,i}=\frac{1}{T_S}\int_{t=iT'_S}^{iT'_S+T_S}\left\{\sum_{n=0}^{N_C-1}d_{n,i}H_1(nf_s)\right\}e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt$$
$$+\frac{1}{T_S}\int_{t=iT'_S}^{iT'_S+\tau_{max}-T_G}\left\{\sum_{n=0}^{N_C-1}d_{n,i}\int_{\tau=T_G}^{\tau_{max}}h_2(\tau)\right.$$

$$\times g(t-\tau-iT'_S)e^{\frac{-j2\pi n\tau}{T_S}}d\tau\Bigg\}e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt$$
$$+\frac{1}{T_S}\int\limits_{t=iT'_S+\tau_{max}-T_G}^{iT'_S+T_S}\Bigg\{\sum_{n=0}^{N_C-1}d_{n,i}H_2(nf_s)\Bigg\}$$
$$e^{\frac{j2\pi(n-l)(t-iT'_S)}{T_S}}dt+\hat{d}_{l,i}^{\mathrm{ISI}} \quad (12)$$

In the time-invariant channel, the additive ICI-CTC is completely removed, thus the demodulated symbol from Eq. (8) can be rewritten as

$$\hat{d}_{l,i}=\hat{d}_{l,i}^{\mathrm{U}}+\hat{d}_{l,i}^{\mathrm{ICI-CIG}}+\hat{d}_{l,i}^{\mathrm{ISI}} \quad (13)$$

The $\hat{d}_{l,i}^{\mathrm{U}}$ can be picked out from the first three terms of Eq. (12) by setting $n=l$. Furthermore, we change the integration bounds with respect to t to omit the index i. The result is

$$\hat{d}_{l,i}^{\mathrm{U}}=d_{l,i}\Bigg\{H_1(lf_s)+\frac{1}{T_S}\int\limits_{t=0}^{\tau_{max}-T_G}\int\limits_{\tau=T_G}^{\tau_{max}}h_2(\tau)$$
$$\times g(t-\tau)e^{\frac{-j2\pi l\tau}{T_S}}d\tau dt$$
$$+\frac{T_S+T_G-\tau_{max}}{T_S}H_2(lf_s)\Bigg\} \quad (14)$$

To simplify the expression of $\hat{d}_{l,i}^{\mathrm{U}}$, we denote:

$$\alpha=\frac{T_S+T_G-\tau_{max}}{T_S} \quad (15)$$

and

$$\eta_l=\frac{1}{T_S}\int\limits_{t=0}^{\tau_{max}-T_G}\int\limits_{\tau=T_G}^{\tau_{max}}h_2(\tau)g(t-\tau)e^{\frac{-j2\pi l\tau}{T_S}}d\tau dt \quad (16)$$

η_l is simply a constant factor, which depends on the sub-carrier index l. In a further analysis of η_l, the integration with respect to τ in Eq. (16) is separated in two intervals. The first interval is $t+T_G<\tau\leq\tau_{max}$, where it is easy to verify, that $g(t-\tau)=0$. The second interval is $T_G<\tau\leq t+T_G$, where $g(t-\tau)=1$. This yields

$$\eta_l=\frac{1}{T_S}\int\limits_{t=0}^{\tau_{max}-T_G}\Bigg\{\int\limits_{\tau=T_G}^{t+T_G}h_2(\tau)e^{-j2\pi lf_s\tau}d\tau\Bigg\}dt \quad (17)$$

Observing the Eq. (17), we see that η_l can be derived from $h_2(\tau)$, T_G, τ_{max} and T_S. Moreover, when $\tau_{max}-T_G\ll T_S$ then η_l is relatively small. With the definitions of α and η_l, the useful symbol becomes:

$$\hat{d}_{l,i}^{\mathrm{U}} \;=\; d_{l,i}\Bigg\{H_1(lf_s)+\alpha H_2(lf_s)+\eta_l\Bigg\} \quad (18)$$

The autocorrelation of $\hat{d}_{l,i}^{\mathrm{U}}$ is used to calculate the useful power P_{U}. While deriving the expression of the useful power, we employ the characteristics of the WSSUS channel model, which has the multi-path profile or the delay power spectrum of the channel defined in [2] as follows

$$\mathrm{E}\{h^*(\tau_1,t)h(\tau_2,t+\Delta t)\}=\rho(\tau_1,\Delta t)\delta(\tau_1-\tau_2) \quad (19)$$

Noting that the channel is time-invariant, therefore the multi-path profile of the channel is replaced by:

$$\mathrm{E}\{h^*(\tau_1)h(\tau_2)\}=\rho(\tau_1)\delta(\tau_1-\tau_2) \quad (20)$$

The final result of the useful power calculation is given as follows:

$$P_{\mathrm{U}}=E_S\Bigg\{\int\limits_{\tau=0}^{T_G}\rho(\tau)d\tau+\alpha^2\int\limits_{\tau=T_G}^{\tau_{max}}\rho(\tau)d\tau$$
$$+\frac{2\alpha}{T_S}\int\limits_{\tau=0}^{\tau_{max}-T_G}\int\limits_{\tau=T_G}^{t+T_G}\rho(\tau)d\tau dt$$
$$+\frac{1}{T_S^2}\int\limits_{t=0}^{\tau_{max}-T_G}\int\limits_{t'=0}^{\tau_{max}-T_G}\int\limits_{\tau=T_G}^{\min\{\mathrm{t+T_G,t'+T_G}\}}\rho(\tau)d\tau dt'dt\Bigg\}$$
$$(21)$$

The expression of $\hat{d}_{l,i}^{\mathrm{ICI-CIG}}$ can be picked out from the first three terms of Eq. (12) by setting $n\neq l$. Since the channel is assumed as time-invariant, the first term of Eq. (12), for $n\neq l$, vanishes. In addition, the integration bounds with respect to t are changed in order to omit the presence of the index i in the integration bounds. Finally, the second term and the third term express the $\hat{d}_{l,i}^{\mathrm{ICI-CIG}}$ and become

$$\hat{d}_{l,i}^{\mathrm{ICI-CIG}} \;=\; \frac{1}{T_S}\sum_{n=0,n\neq l}^{N_C-1}d_{n,i}\Bigg\{\int\limits_{t=0}^{\tau_{max}-T_G}\Bigg[\int\limits_{\tau=T_G}^{\tau_{max}}h_2(\tau)$$
$$\times g(t-\tau)e^{\frac{-j2\pi n\tau}{T_S}}d\tau\Bigg]e^{\frac{j2\pi(n-l)t}{T_S}}dt$$
$$+\int\limits_{t=\tau_{max}-T_G}^{T_S}H_2(nf_s)e^{\frac{j2\pi(n-l)t}{T_S}}dt\Bigg\} \quad (22)$$

As discussed earlier in section II, the ICI-CIG consists of two parts. One part results from the FFT of $y_{i_2}(t)$ in area I in fig. 1.f and is the first term of Eq. (22) (see the integration interval with respect to t), the other part results from the FFT of $y_{i_2}(t)$ in area II in Fig. 1.f and is the second term of Eq. (22). In the first term of Eq. (22), we change the integration bounds with respect to τ as explained in Eq. (17) to have $g(t-\tau)=0$ for

$t+T_G < \tau \le \tau_{max}$ and $g(t-\tau) = 1$ for $T_G < \tau \le t+T_G$. We get the result:

$$\hat{d}_{l,i}^{\mathrm{ICI-CIG}} = \frac{1}{T_S} \sum_{n=0, n\neq l}^{N_C-1} d_{n,i} \Bigg\{ \int_{t=0}^{\tau_{max}-T_G} \Big[\int_{\tau=T_G}^{t+T_G} h_2(\tau) e^{\frac{-j2\pi n\tau}{T_S}} e^{\frac{j2\pi(n-l)t}{T_S}} d\tau \Big] dt + \int_{t=\tau_{max}-T_G}^{T_S} H_2(nf_s) e^{\frac{j2\pi(n-l)t}{T_S}} dt \Bigg\} \quad (23)$$

The autocorrelation of $\hat{d}_{l,i}^{\mathrm{ICI-CIG}}$ is used to calculate the ICI-CIG power. The final result is given as follows:

$$P_{\mathrm{ICI-CIG}} = \frac{E_S}{T_S^2}(\tau_{max} - T_G) \int_{t=0}^{\tau_{max}-T_G} \int_{\tau=T_G}^{t+T_G} \rho(\tau) d\tau - \frac{E_S}{T_S^2} \int_{t'=0}^{\tau_{max}-T_G} \int_{t=0}^{\tau_{max}-T_G} \int_{\tau=T_G}^{\min\{\mathrm{t+T_G, t'+T_G}\}} \rho(\tau) d\tau dt dt' + \frac{E_S}{T_S^2}(T_G - \tau_{max} + T_S) \int_{t=0}^{\tau_{max}-T_G} \int_{\tau=t+T_G}^{\tau_{max}} \rho(\tau) d\tau dt \quad (24)$$

If $\tau_{max} - T_G \ll T_S$, then the first and the second terms of Eq. (24) are negligible. $P_{\mathrm{ICI-CIG}}$ is well approximated as follows:

$$P_{\mathrm{ICI-CIG}} \approx \frac{E_S}{T_S} \int_{t=0}^{\tau_{max}-T_G} \int_{\tau=t+T_G}^{\tau_{max}} \rho(\tau) d\tau dt \quad (25)$$

The equation (24) states that *the ICI-CIG power depends on the tail outside the guard interval of the multi-path-channel profile.* If $\tau_{max} - T_G$ is relatively small in comparison with the OFDM symbol duration, then the ICI-CIG power is approximately equal to the ISI power, which is obtained later in Eq. (29).

IV. Analysis of ISI

The effect of intersymbol interference is that the current symbol $\hat{d}_{l,i}$ is influenced by a number of previous and following symbols in the same sub-carrier. ISI only occurs, if the guard length is insufficient. In this case, the second term of Eq. (7) is rewritten as:

$$\hat{d}_{l,i}^{\mathrm{ISI}} = \frac{1}{T_S} \sum_{i'=-\infty, i'\neq i}^{+\infty} \int_{t=iT'_S}^{iT'_S+T_S} \Bigg\{ \sum_{n=0}^{N_C-1} d_{n,i'} \int_{\tau=0}^{\tau_{max}} h_2(\tau) g(t-\tau-i'T'_S) e^{\frac{-j2\pi n\tau}{T_S}} d\tau \Bigg\} e^{\frac{j2\pi[n(t-i'T'_S)-l(t-iT'_S)]}{T_S}} dt \quad (26)$$

The channel is assumed to be causal and non-zero solely in the time interval $\tau \in [0, T_S]$, i.e. the channel impulse response is not longer than an OFDM symbol duration. In addition, the basic impulse is time-limited as in Eq. (2). This leads to the conclusion that only the previous OFDM symbol is involved in the current OFDM symbol. Therefore, in Eq. (26) only the time index $i' = i-1$ is taken into account. Furthermore, we change the bounds of the integrations so that the basic impulse $g(t-\tau-iT'_S+T'_S) = 1$, for all τ and t in the integration intervals. We get:

$$\hat{d}_{l,i}^{\mathrm{ISI}} = \frac{1}{T_S} \int_{t=iT'_S}^{iT'_S+\tau_{max}-T_G} \Bigg\{ \sum_{n=0}^{N_C-1} d_{n,i-1} \int_{\tau=T_G+t-iT'_S}^{\tau_{max}} h_2(\tau) e^{\frac{-j2\pi n\tau}{T_S}} d\tau \Bigg\} e^{\frac{j2\pi[(n-l)(t-iT'_S)+nT'_S]}{T_S}} dt \quad (27)$$

The mean square value of $\hat{d}_{l,i}^{\mathrm{ISI}}$ expresses the ISI power. After some manipulations, we obtain

$$P_{\mathrm{ISI}}(l,i) = \frac{E_S}{T_S^2} \sum_{n=0}^{N_C-1} \int_{t=iT'_S}^{iT'_S+\tau_{max}-T_G} \int_{t'=iT'_S}^{iT'_S+\tau_{max}-T_G} \int_{\tau=\max\{\mathrm{T_G+t-iT'_S, T_G+t'-iT'_S}\}}^{\tau_{max}} \rho(\tau, t-t') d\tau \times e^{\frac{j2\pi(n-l)(t-t')}{T_S}} dt dt' \quad (28)$$

The time index i in Eq. (28) can be omitted by changing the integration bounds with respect to t and t'. Additionally, we exchange summation and integration. Taking the orthogonality of the summation referring to the index n into account and assuming $N_C = N_{FFT}$, the ISI power becomes

$$P_{\mathrm{ISI}} = \frac{E_S}{T_S} \int_{t=0}^{\tau_{max}-T_G} \int_{\tau=t+T_G}^{\tau_{max}} \rho(\tau) d\tau dt \quad (29)$$

If $N_C < N_{FFT}$, then P_{ISI} must be multiplied with a factor $K = N_C/N_{FFT}$. From Eq. (29) we conclude that *the ISI power depends on the tail outside the guard interval of the multi-path channel profile, the system parameters T_G, T_S and the symbol energy E_S. Interestingly, for the WSSUS channel, it is independent of the sub-carrier index l and the time index i.*

V. Numerical results

Basically, the OFDM parameters of the considered system were taken from HIPERLAN/2 specified in [4]. The indoor channel model A where the average power declines exponentially as described in [1] was taken into consideration. However the sampling duration of the system

($t_a = 1/B = 50$ns) does not match to the minimum tap delay (10 ns). Therefore, based on the existing coefficients of this channel, all the coefficients of our channel model which do not coincide with HIPERLAN/2 coefficients, are interpolated.

Due to the fact that two data symbols $(d_{l,i}, d_{l,i-1})$ of two adjacent OFDM symbols are statistically independent, $\hat{d}_{l,i}^{\mathrm{ICI-CIG}}$ and $\hat{d}_{l,i}^{\mathrm{ISI}}$ are also statistically independent. Therefore the total interference power is the sum of ICI-CIG and ISI powers:

$$P_{\mathrm{I}} = P_{\mathrm{ICI-CIG}} + P_{\mathrm{ISI}} \tag{30}$$

where $P_{\mathrm{ICI-CIG}}$ and P_{ISI} are computed from Eqs. (24) and (29), respectively. There is of course no contribution of $P_{\mathrm{ICI-CTC}}$ because the channel is time-invariant. The simulation result of the interference power P_{I}' is the mean square value of the difference

$$\hat{d}_{l,i}^{\mathrm{I}} = \hat{d}_{l,i} - \hat{d}_{l,i}^{\mathrm{U}} \tag{31}$$

where $\hat{d}_{l,i}^{\mathrm{U}}$ is computed from Eq. (15) under the assumption that the channel coefficients are known. The simulation result of the interference power P_{I}' is obtained by evaluating the estimation

$$P_{\mathrm{I}}' = \mathrm{E}\{(\hat{d}_{l,i}^{\mathrm{I}})^* \hat{d}_{l,i}^{\mathrm{I}}\} \tag{32}$$

In order to simulate the static channel with uncorrelated scattering, we carried out numerous experiments, in each experiment the channel coefficient for the k-th path is given as follows

$$h_k^s = a_k + jb_k \tag{33}$$

where a_k and b_k are two realization coefficients of two zero-mean statistically independent gaussian random variables with variances being $h_k^2/2$ (this is also $\rho_k/2$). h_k is the real channel coefficient of path k taken from [1] and h_k^s is our complex channel model coefficient of the s-th experiment. It can be easily proved that $\mathrm{E}\{(h_k^s)^* h_k^s\} = \rho_k$ and $\mathrm{E}\{(h_k^s)^* h_l^s\} = 0$ for $k \neq l$, if the number of experiments is large enough.

As shown in Fig. 2, comparing the simulation results obtained by one experiment with the theoretical result, we see that the simulation result fluctuates around the calculation result. However, with 100 experiments for instance, the simulation result agrees very well with the calculation result. It is interesting to compare quantitatively the ISI power with the ICI-CIG power in (24) as depicted in Fig. 3. In this case, $\tau_{max} - T_G$ is relatively small in comparison with the OFDM symbol duration T_S, thus the ICI-CIG power is approximately equal to the ISI power.

VI. Conclusions

By truncating the channel impulse response into two parts, one being inside the guard interval, the other outside, we have derived mathematical expressions of ICI and ISI in case of time-invariant channels. Agreement between the numerical simulations and the theoretical calculations demonstrates the validity of our method. The analysis results obtained for the time-varying channel is in preparation for further publication.

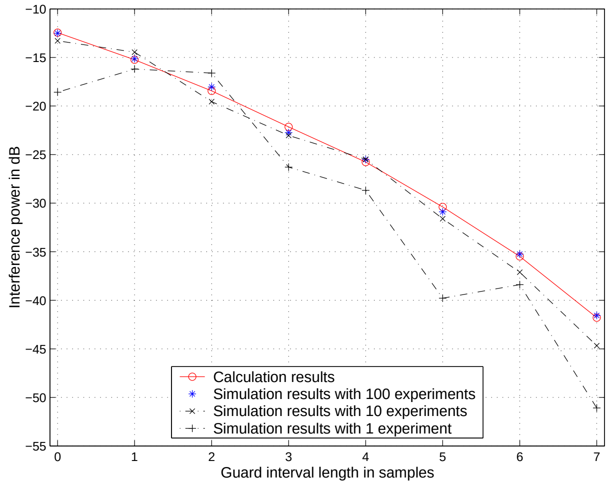

Fig. 2. Comparison of the theoretical calculation of interference power with the simulation results on a time-invariant channel with uncorrelated scattering.

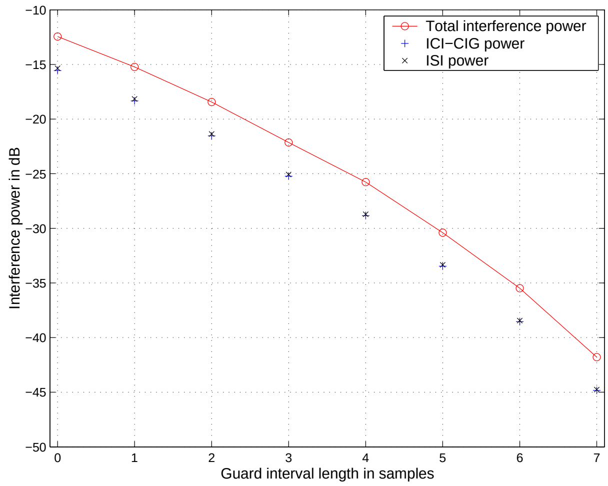

Fig. 3. Illustration of ISI power and ICI-CIG power.

References

[1] Medbo, J.; Schramm, P. *Channel Model for HIPERLAN/2 in Different Indoor Scenarios.* ETSI EP BRAN 3ERI085B, 30 March 1998.

[2] Proakis, J.G. *Digital Communications.* 3. edition, New York: McGraw-Hill, 1995.

[3] Russell, M.; Stüber, G. L. *Interchannel Interference Analysis of OFDM in a Mobile Environment.* Proceedings of VTC 1995, p. 820-824.

[4] ETSI DTS/BRAN-0023003 *HIPERLAN Type 2 Technical Specification; Physical(PHY) layer.* 1999.