Capacity-Delay-Error-Boundaries: A Composable Model of Sources and Systems

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March 09, 2015

Information vs. queueing theory

Information theory focuses on averages and asymptotic limits of

- the data rate of a source,
- the capacity of a channel.

It does, however, not consider delays that are due to their variability.

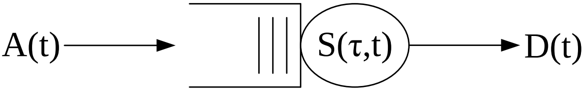
Unlike **queueing theory** that considers

- the burstiness of sources and statistical multiplexing,
- delays and loss that can be traded for capacity,

but assumes statistics of sources.

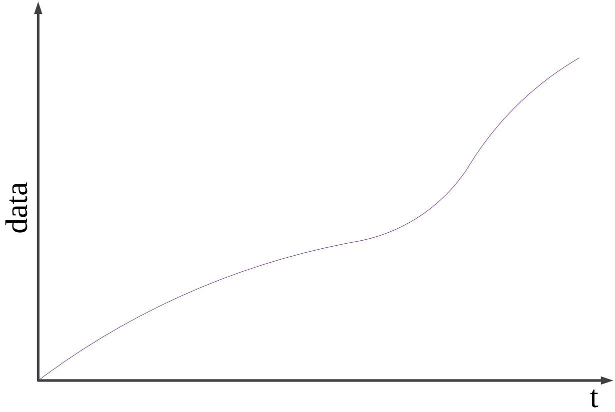
Outline

Network Calculus [Chang '00]



$$A \otimes S(t) := \inf_{\tau \in [0,t]} \{A(\tau) + S(\tau, t)\} \leq D(t)$$

Statistical network calculus [CLB '06]



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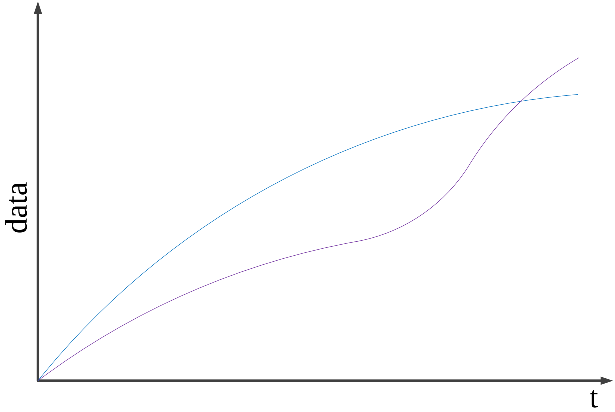

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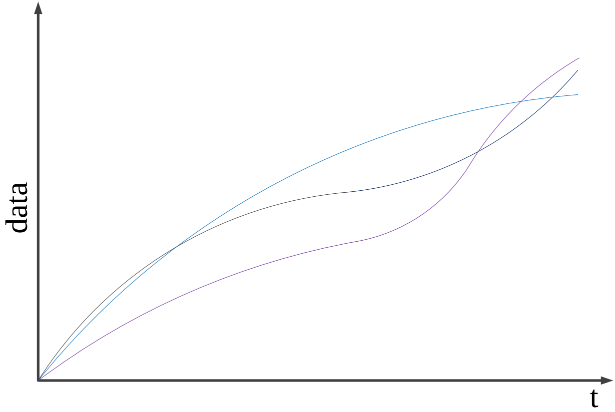

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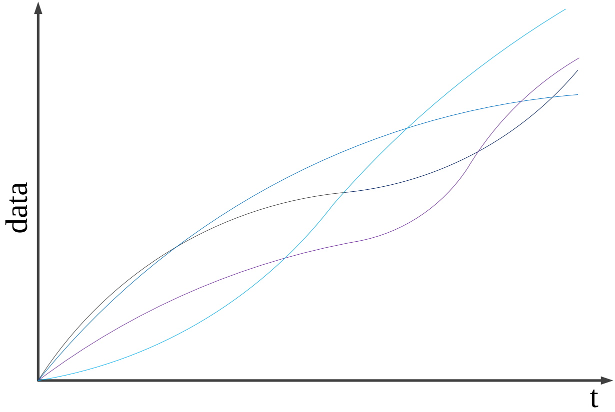

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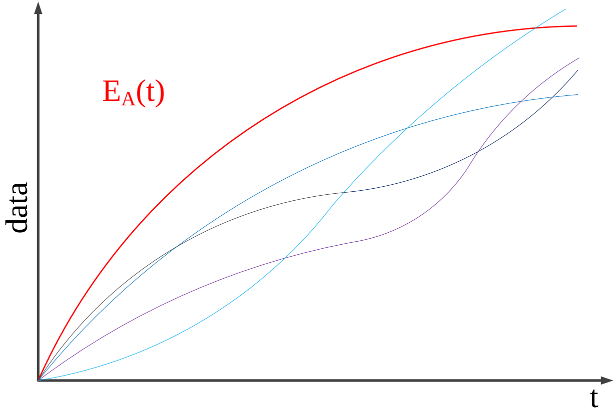
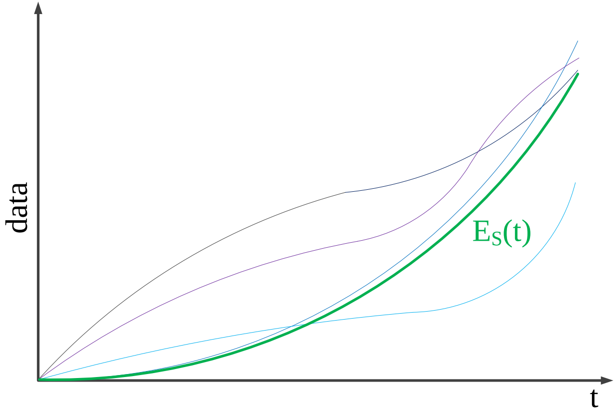
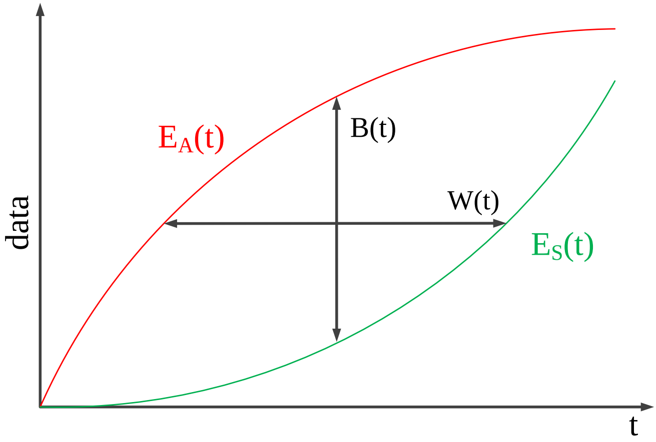


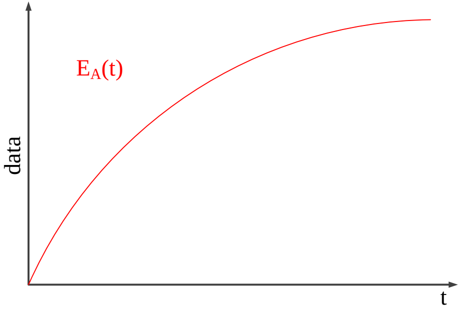
Statistical network calculus [LFL '14]



Performance bounds



For a constant rate server with capacity c



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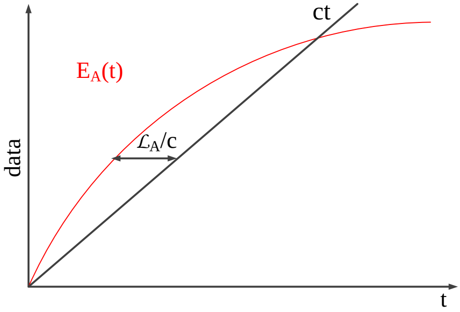

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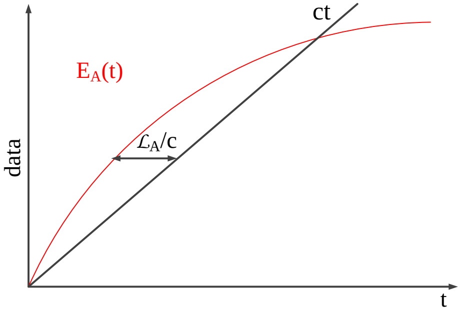

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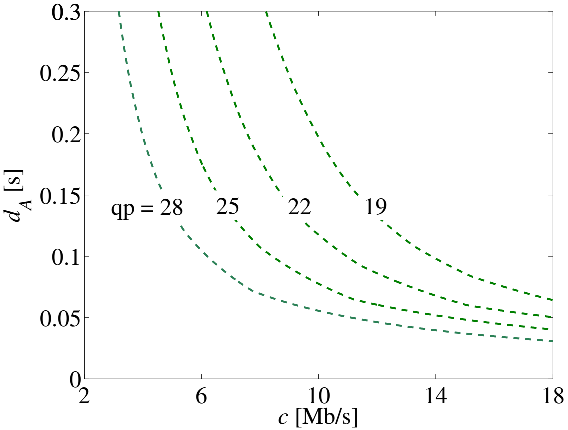
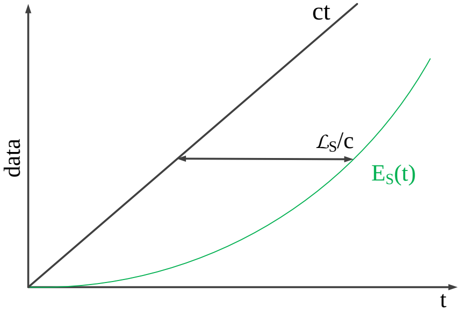
$$d_A = \mathcal{L}_A(c)/c \ .$$

Source



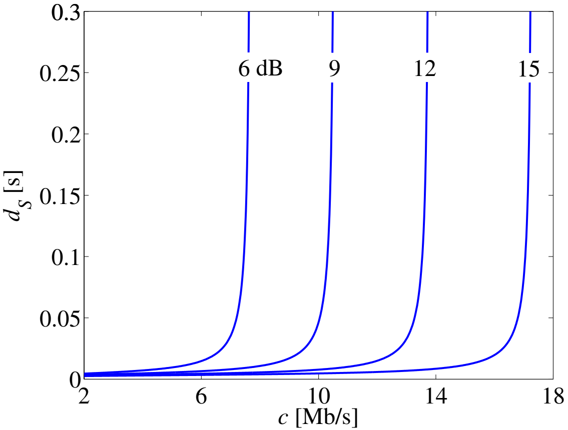
For a constant arrival rate c



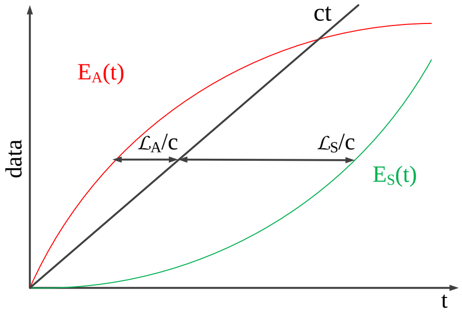
$$\mathcal{L}_S(c) := \sup_{t \geq 0}\{ct - E_S(t)\}$$

$$d_S = \mathcal{L}_S(c)/c \ .$$

System [ALB '13]



Additivity of the Capacity-Delay-Error Model



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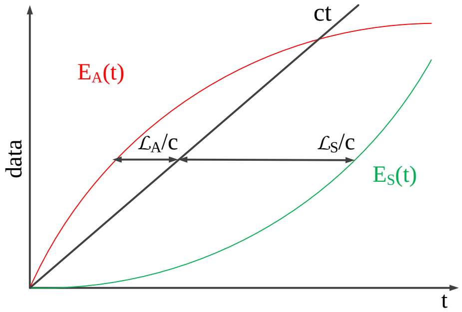

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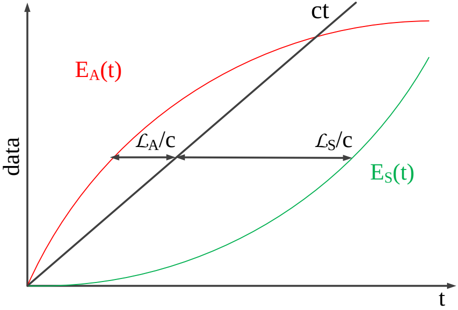

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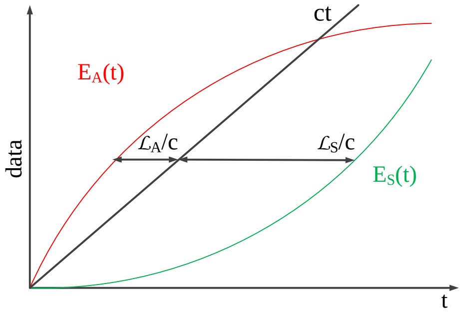

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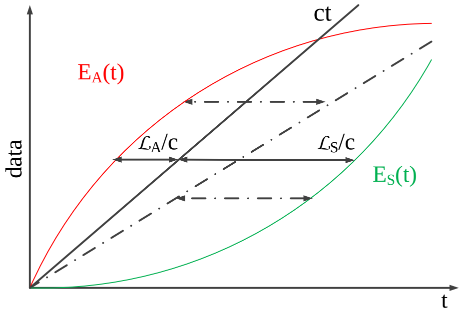

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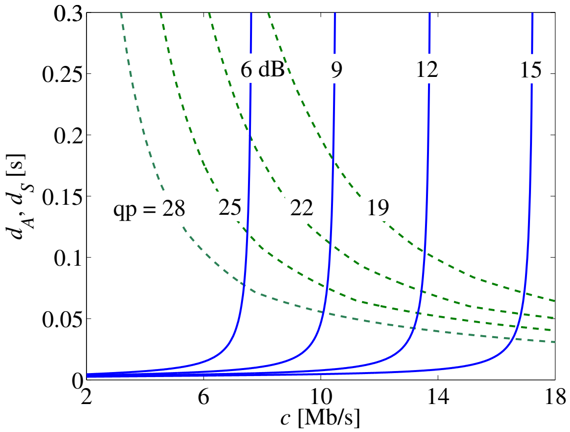
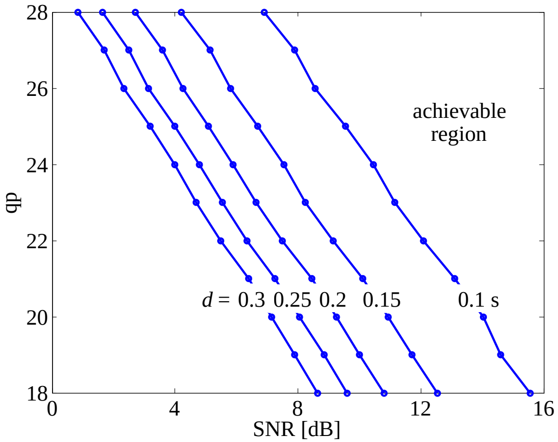
Composite systems



Feasible Operating Points



Conclusions

Legendre transforms of $E_A(t)$ and $E_S(t)$

- additive backlog and delay bounds
- permits analyzing sources and channels separately
- have the interpretation of a capacity-delay-error-tradeoff
- model includes
  - memoryless sources, Markov sources,
  - Huffman, Shannon, Lempel-Ziv coders,
  - discrete memoryless channels
  - block coding, e.g., BCH codes
- can be a step towards a unified theory ...